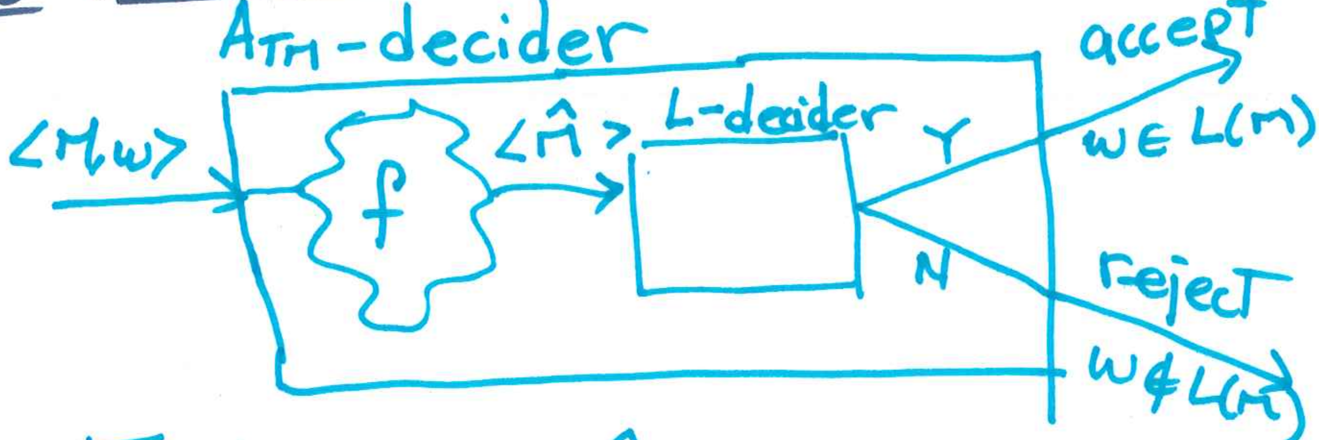
Rices' THEOREM: $A = A_{TM}$, $B = L$



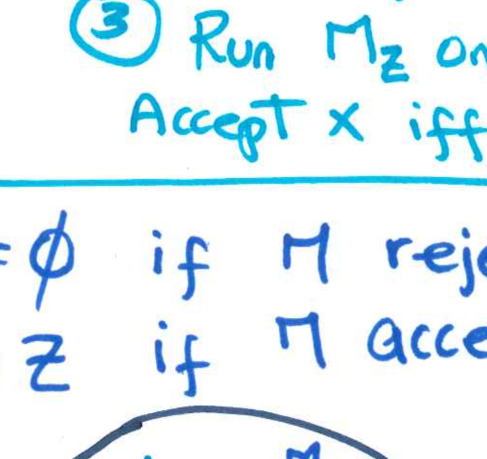
Template for $\hat{M}$

$\hat{M}$

ON input x
1. Simulate M on w
2. If M rejects, then halt and reject
   else (M accepts w)
3. Run M_Z on X
   Accept x iff M_Z accepts

$L(\hat{M}) = \emptyset$ if M rejects w $(\langle \hat{M} \rangle \in \overline{L})$

$L(\hat{M}) = Z$ if M accepts w $(\langle \hat{M} \rangle \in L)$



Assuming that $\langle M_\phi \rangle \in \overline{L}$

Fact: If $A \leq_m B$ and B is decidable, then A is decidable.

Fact: If $A \leq_m B$ and A is undecidable, then B is undecidable.

Fact: If $A \leq_m B$ and B is Turing-Recognizable then A is Turing-Recognizable.

Fact: If $A \leq_m B$ and A is not Turing-Recognizable then B is not Turing recognizable.

$$L = \{\langle M \rangle \mid L(M) \neq \phi\}$$

$\hat{M}$

> 1. Simulate M on w
> 2. if M rejects, then halt and reject
>
> else (M accepts w)
>
> 3. Simulate M_{Σ^*} on x and accept iff M_{Σ^*} accepts

$L(\hat{M}) = \phi$ if M does not accept w

$L(\hat{M}) = \Sigma^*$ if M accepts w

In other words:

$$\langle M, w \rangle \in A_{TM} \iff \langle \hat{M} \rangle \in L$$

$f(\langle M, w \rangle) = \langle \hat{M} \rangle$, so f is a Reduction.

L is undecidable.