Final Exam: Solution
Artificial Intelligence (Spring 2007)

Question 1 English x First Order Logic(FOL) (36 points)

Check the answer with True or False. For sentences in English make your judgment of the meaning of the sentence, i.e., you may want to translate it in FOL to conclude.

1. **(4 points)** "Bert and Ernie are brothers" is equivalent to "Bert is a brother and Ernie is a brother"

   True[ ]　　　　　　False[X]

   The second sentence does not assert that they are brothers to each other.

2. **(4 points)** " p and q are not both true" is equivalent to " p and q are both not true"

   True[ ]　　　　　　False[X]

   The first one says "$\neg(p \wedge q)$" equivalent to "$\neg p \vee \neg q$". The second says "$\neg p \wedge \neg q$" equivalent to "$\neg(p \vee q)$".

3. **(4 points)** "Neither p nor q" is equivalent to "both p and q are false"

   True[X]　　　　　　False[ ]

   English Neither, nor means "$\neg p \wedge \neg q$"

4. **(4 points)** "Not all A's are B's" is equivalent to "$\exists x\ (A(x) \wedge \neg B(x))$"

   True[X]　　　　　　False[ ]

   It means, some A's are not B's.

5. **(4 points)** "Men and women are welcome to apply." is equivalent to "$\forall x\ [(M(x) \wedge W(x)) \Rightarrow Apply(x)]$"

   True[ ]　　　　　　False[X]

   The FOL sentence says that everything that is *both* a man and a woman is welcome to apply, obviously not what is meant. The disjunction would give a correct interpretation.

Questions 6 to 9: Say Attract is a relation from x to y, i.e., A(x,y) says that x attracts y.

6. **(4 points)** "Everything attracts something", where "something" means "something or other", is equivalent to "$\forall x \; \exists y \; A(x, y)$"

True[X] False[ ]

7. **(4 points)** "Something is attracted by everything", where "something" means "something in particular", is equivalent to "$\exists y \; \exists x \; A(x, y)$"

True[ ] False[X]

The second quantifier should have been $\forall x$ to represent everything.

8. **(4 points)** "Everything is attracted by something" ", where "something" means "something or other", is equivalent to "$\exists x \; \forall y \;\; A(x, y)$"

True[X] False[ ]

9. **(4 points)** "Something attract everything", where "something" means "something in particular", is equivalent to "$\exists x \; \exists y \; A(x, y)$"

True[ ] False[X]

The second quantifier should have been $\forall y$ to represent everything

Question 2 KB and Goal with Resolution method (35 points)

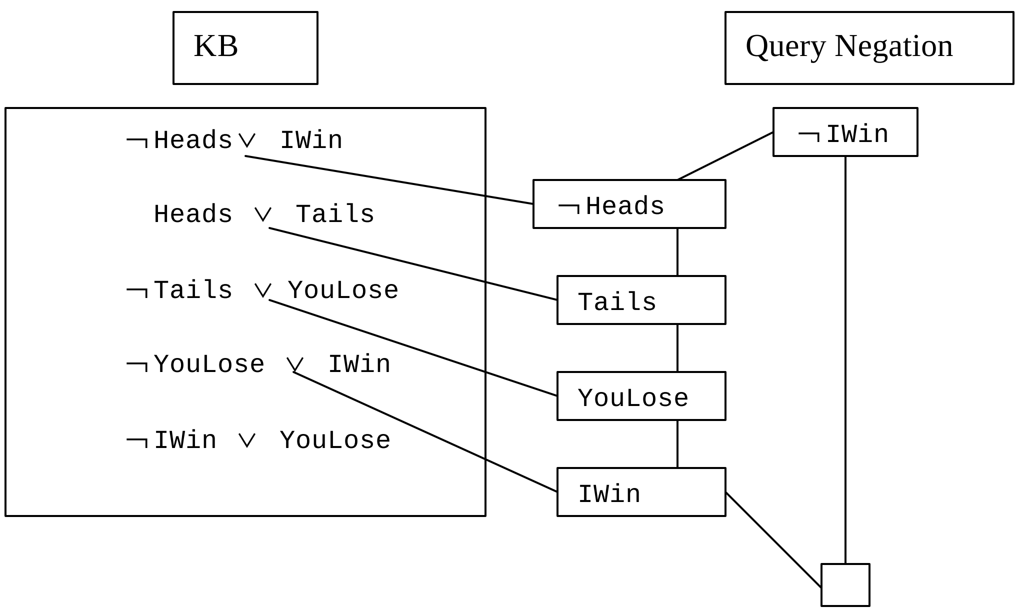
1. From the sentence "Heads I win, tails you lose," prove using the resolution method that "I win." More precisely,

a. **(7 points)** First build the KB, from the sentence "Heads I win, tails you lose," using the true or false variables `Heads, Tails, IWin, YouLose` and write the sentence in terms of disjunctions clauses. Add to KB the general knowledge that the outcome of a coin toss must be `Head` or `Tails` and the general knowledge that if `YouLose` then `IWin` and, if `IWin` then `YouLose.`

"Heads I win, tails you lose."
`(Heads => IWin)` or in CNF `(¬Heads ∨ IWin)`
`(Tails => YouLose)` or in CNF `(¬Tails ∨ YouLose)`

Add some general knowledge axioms about coins, winning, and losing:
`(Heads v Tails)`
`(YouLose => IWin)` or in CNF `(¬YouLose ∨ IWin)`
`(IWin => YouLose)` or in CNF `(¬IWin ∨ YouLose)`

b. **(7 points)** Prove the Goal sentence: "`IWin`" via the Resolution method.



2. From the statement:

Tony, Shi-Kuo and Ellen belong to the Hoofers Club. Every member of the Hoofers Club is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Ellen dislikes whatever Tony likes and likes whatever Tony dislikes. Tony likes rain and snow.

Prove via the resolution method and unification (where needed) that "Ellen is a mountain climber but not a skier". More precisely

a. **(7 points)** First translate the sentences above into FOL Sentences

Let `S(x)` mean `x` is a skier, `M(x)` mean `x` is a mountain climber, and `L(x,y)` mean `x` likes `y`, where the domain of the first variable is Hoofers Club members, and the domain of the second variable is snow and rain. Translate the above English sentences into FOL using quantifiers, ∃,∀ where appropriate.

R (not the only way to write it):

1. `(∀x) S(x) ∨ M(x)` - "Every member of the Hoofers Club is either a skier or a mountain climber or both."
2. `¬ (∃x) M(x) ∧ L(x, Rain)` - "No mountain climber likes rain"
3. `(∀x) S(x) => L(x, Snow)` - "all skiers like snow"
4. `(∀y) L(Ellen, y) => ¬L(Tony, y)` - "Ellen likes whatever Tony disklikes"
5. `(∀y) ¬L(Ellen, y) => L(Tony, y)` - "Ellen dislikes whatever Tony likes"
6. `L(Tony, Rain)` - "Tony likes rain"
7. `L(Tony, Snow)` - "Tony likes snow"
8. Query: `M(Ellen) ∧ ¬S(Ellen)`

Negation of the Query: `¬(M(Ellen) ∧ ¬S(Ellen))`

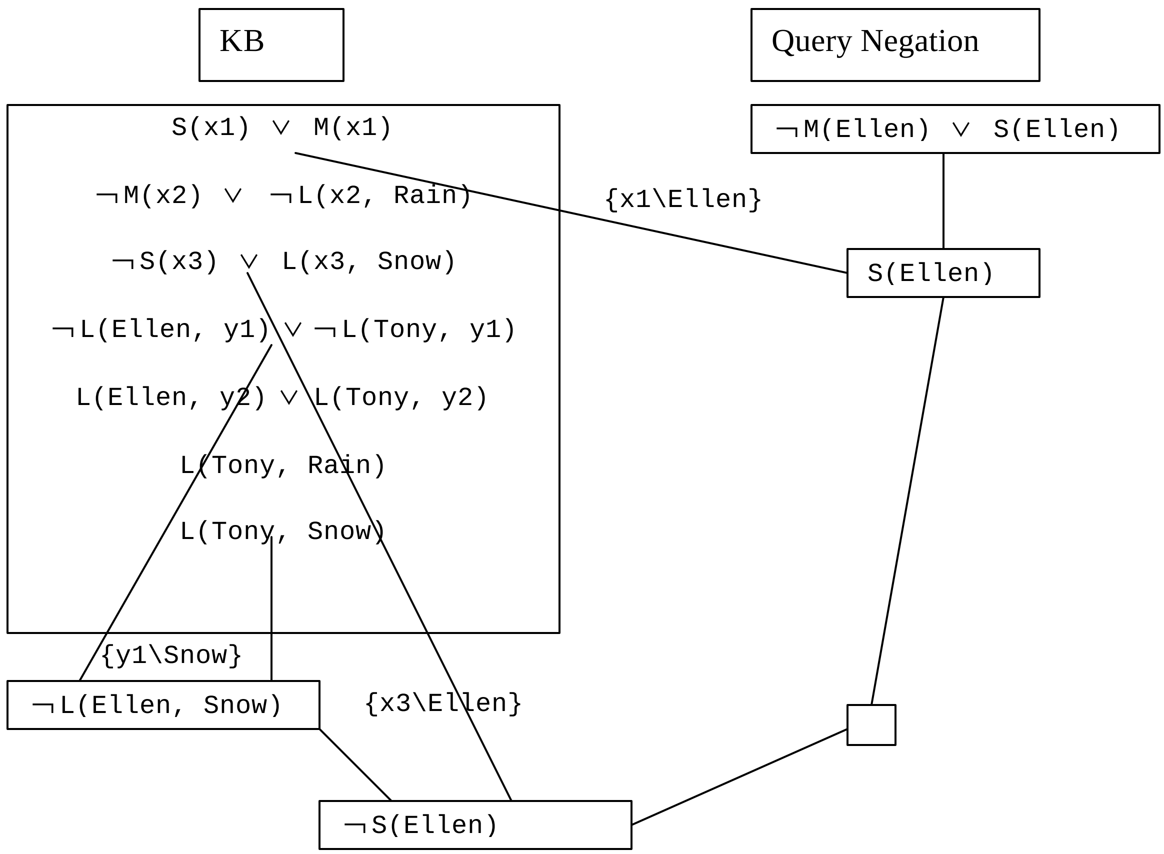
b. **(7 points)** Convert to Clause Forms (in these cases disjunctions)

Conversion to Clause Form

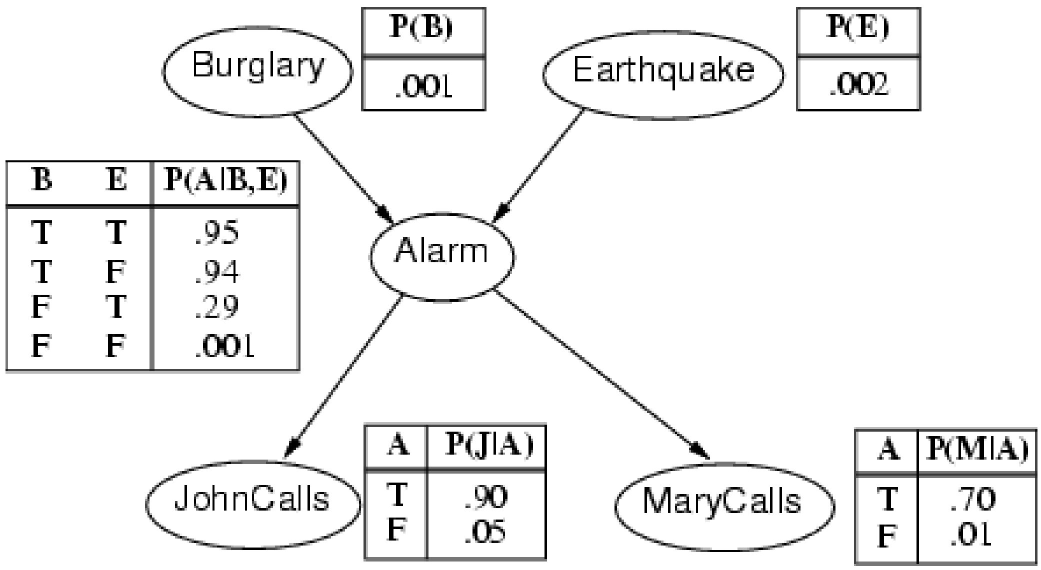
9. `S(x1) ∨ M(x1)`
10. `¬M(x2) ∨ ¬L(x2, Rain)`
11. `¬S(x3) ∨ L(x3, Snow)`
12. `¬L(Ellen, y1) ∨ ¬L(Tony, y1)`
13. `L(Ellen, y2) ∨ L(Tony, y2)`
14. `L(Tony, Rain)`
15. `L(Tony, Snow)`

16. Negation of the Query: ¬M(Ellen) ∨ S(Ellen)

c. **(7 points)** Apply the resolution method with the unification (where needed) to prove the goal sentence: "Ellen is a mountain climber but not a skier." You need to also convert the goal sentence to a Clause Form.



Question 3 Uncertainty Bayesian Nets (29 points)



A Bayesian network, showing both the topology and the conditional probability tables (CPTs). In the CPTs, the letters B, E, A, J and M stand for Burglary, Earthquake, Alarm, John Calls, and MaryCalls, respectively. The independent conditional probability help us to write in a simplified way the joint distribution P(B,E,A,J,M).

1. **(10 points)** Express the joint distribution P(B,E,A,J,M) in terms of the conditional probabilities (and independencies) expressed in the Bayesian Network above.

R: The general formula is $P(x_1,...,x_n) = \prod_{i=1}^{n} P(x_i \mid parents(X_i))$, so in this case becomes $P(B,E,A,J,M) = P(J \mid A)\ P(M \mid A)\ P(A \mid B,E)\ P(B)\ P(E)$

2. We want to know the probability of a Burglary knowing that John Called and Mary Called. More precisely,

   a. **(9 points)** Which variables are hidden? What do you do with hidden variables to compute probabilities?

R: hidden variables are E and A. We sum over the hidden variables.

b. **(10 points)** Calculate P(B|J,M), i.e., calculate P(Burglary=true | JohnCalls=true, MaryCalls=true) with at least two decimal orders.

$$P(B \mid J=T, M=T) = a \sum_{E=F}^{T} \sum_{A=F}^{T} P(B, E, A, J=T, M=T)$$

$$= a \sum_{E=F}^{T} \sum_{A=F}^{T} P(J=T \mid A)\ P(M=T \mid A)\ P(A \mid B, E)\ P(B)\ P(E)$$

$$= a\ P(B) \sum_{e=F}^{T} P(E) \left(\sum_{A=F}^{T} P(A \mid B, E)\ P(J=T \mid A)\ P(M=T \mid A) \right)$$

$$= a\ P(B) \sum_{e=F}^{T} P(E) \left(P(A=T \mid B, E) * 0.9 * 0.7 + P(A=F \mid B=T, E) * 0.05 * 0.01 \right)$$

$$= a\ P(B) \begin{bmatrix} 0.002 * \left(P(A=T \mid B, E=T) * 0.9 * 0.7 + P(A=F \mid B=T, E=T) * 0.05 * 0.01 \right) \\ + 0.998 * \left(P(A=T \mid B, E=F) * 0.9 * 0.7 + P(A=F \mid B=T, E=F) * 0.05 * 0.01 \right) \end{bmatrix}$$

$$= a \begin{pmatrix} 0.001 * (0.00126 * 0.95 + 0.000001 * 0.05 + 0.62874 * 0.94 + 0.000499 * 0.06), \\ 0.999 * (0.00126 * 0.29 + 0.000001 * 0.71 + 0.62874 * 0.001 + 0.000499 * 0.999) \end{pmatrix}$$

$$= a\ (0.00059224, 0.0014919) \quad \Rightarrow a = 1 / (0.00059224 + 0.0014919)$$

$$\approx (0.284, 0.716)$$

Thus $P(B=T \mid J=T, M=T) \approx 0.284$