1. (*30 points*) Let the vector valued function from $\mathbb{R} \to \mathbb{R}^3$ be given by $\vec{r}(t) = \langle 3\cos t, 4t, 3\sin t \rangle$.

(i) Find $\vec{T}(t)$ (unit tangent), $\vec{N}(t)$ (principle unit normal), $\vec{B}(t)$ (unit binormal).

(ii) Find the normal plane (normal to the direction) at time $t = 0$.

2. (*25 points*)
Find the equation of the tangent plane (in $\mathbb{R}^3$) to the graph of

$$f(x, y) = x^2 y$$

at the point

$$(x, y, f(x, y)) = (1, 1, 1).$$

3. (*30 points*) Find the extreme values of

$$f(x, y, z) = x^2 + y^2 + z^2,$$

subject to the constraints

$$x - y = 1 \text{ and } y^2 - z^2 = 1.$$

4. (*30 points*) Evaluate the integral:

$$\int_0^2 \int_0^{z^2} \int_{-y}^{y-z} (2x - y)\, z \, dx \, dy \, dz.$$

5. (*30 points*) Find the volume of the solid *inside* the sphere

$$x^2 + y^2 + z^2 = 16$$

and *outside* the cylinder

$$x^2 + y^2 = 4.$$

You may want to sketch the solid first.

6. (*25 points*) Warning: Do the following carefully. It is tricky.

(i) Draw a picture of the region of integration of $\int_0^1 \int_x^{2x} dy\, dx$.

(ii) Use part (i) to switch the order of integration, ie express the integral in terms of $dx\, dy$.

7. (*15 points*) For a particle moving in space, $\mathbb{R}^3$, assume that the velocity is always perpendicular to position. Prove that the particle moves on a sphere centered at the origin, $(0, 0, 0)$.

8. (*15 points*) Is there a function $f : \mathbb{R}^2 \to \mathbb{R}$ with partial derivatives (on the whole plane) $f_x(x, y) = 3x + 2y$ and $f_y(x, y) = 3y$? Why or why not? For any credit you must rigorously defend your point.