

Homework 11 for M312, Section 30353
due Wednesday, November 20, 2013

1. (10 pts) Exercise 8.2.2 (p.450)
2. (10 pts) Exercise 8.2.3 (p.450)
3. (10 pts) Exercise 8.2.4 (p.450)
4. (10 pts) Exercise 8.2.6 (p.450)
5. (10 pts) Exercise 8.2.7 (p.450)
6. (20 pts) Exercise 8.2.26 (p. 452)

Mean-Value Theorem for Harmonic Functions. Assume that u is a harmonic function (that is u is C^2 and $\Delta u = 0$) in the unit disc $D = \{(x, y) : x^2 + y^2 \leq 1\}$. Then

$$u(0, 0) = \frac{1}{2\pi} \int_{\partial D} u \, ds.$$

7. (15 pts) Verify this theorem for $u(x, y) = x^3 - 3xy^2$ (check that it is harmonic and that the formula holds) .
8. (15 pts) Use this theorem for $u(x, y) = e^x \cos y$ to compute

$$\int_0^{2\pi} e^{\cos t} \cos(\sin t) dt.$$

9. (extra credit, 20 pts) Use this theorem to compute

$$\int_0^{2\pi} \log(5 - 4 \cos t) dt.$$