MATH 343 Summer I 2013 15 June 5^{th} ,2013
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Quiz#3 Name: Enrique Areyan

You have 20 minutes to finish the following 2 problems.

1. (*8 points*) Use the method of undetermined coefficients to determine y_p for the following ODEs. Don't solve for the coefficients.

$$y''' - 2y'' + y' = t^3 + 2e^t$$

$\underline{y_h}$: $r^3 - 2r^2 + r = 0 \iff r(r^2 - 2r + 1) = 0 \iff r(r-1)^2 = 0$ $\boxed{r_1 = 0;\ r_{2,3} = 1}$

$$\boxed{y_h = C_1 + C_2 e^t + C_3 t e^t}$$

So, the guess for particular is:

$$\boxed{y_p = \underbrace{t(At^3 + Bt^2 + Ct + D)}_{\text{Corresponds to } t^3} + \underbrace{Et^2 e^t}_{\text{corresponds to } 2e^t}}$$

$$y^{(4)} + 4y'' = sin(2t) + te^t + 4$$

$\underline{y_h}$: $r^4 + 4r^2 = 0 \iff r^2(r^2 + 4) = 0 \Rightarrow \boxed{r_{1,2} = 0;\ r_{3,4} = \pm 2i}$

$$\boxed{y_h = C_1 + tC_2 + C_3\cos(2t) + C_4\sin(2t)}$$

So, the guess for particular is:

$$\boxed{y_p = \underbrace{t^2 A}_{\text{corresponds to } 4} + \underbrace{[B\cos(2t) + C\sin(2t)]t}_{\text{corresponds to } \sin(2t)} + \underbrace{(Dt + E)e^t}_{\text{corresponds to } te^t}}$$

2. (*7 points*) Use Variation of Parameters to solve the following ODE.

$$y''' + y' = sec(t)$$

y_h: $r^3 + r = 0 \Leftrightarrow r(r^2+1) = 0 \Leftrightarrow r_1 = 0$; $r_{2,3} = \pm i$

$\boxed{y_h = C_1 + C_2\cos(t) + C_3\sin(t)}$

the solution is $\boxed{y = u_1y_1 + u_2y_2 + u_3y_3}$

Let $y_1 = 1$; $y_2 = \cos(t)$; $y_3 = \sin(t)$

then $W(1, \cos(t), \sin(t)) = \begin{vmatrix} 1 & \cos(t) & \sin(t) \\ 0 & -\sin(t) & \cos(t) \\ 0 & -\cos(t) & -\sin(t) \end{vmatrix} = 1$

$W_1 = \begin{vmatrix} 0 & \cos(t) & \sin(t) \\ 0 & -\sin(t) & \cos(t) \\ 1 & -\cos(t) & -\sin(t) \end{vmatrix} = +\cos^2(t) + \sin^2(t) = 1$

$W_2 = \begin{vmatrix} 1 & 0 & \sin(t) \\ 0 & 0 & \cos(t) \\ 0 & 1 & -\sin(t) \end{vmatrix} = -\cos(t)$; $W_3 = \begin{vmatrix} 1 & \cos(t) & 0 \\ 0 & -\sin(t) & 0 \\ 0 & -\cos(t) & 1 \end{vmatrix} = -\sin(t)$

So, $u_i' = \frac{W_i \cdot g}{W}$, where $i = 1, 2, 3$ and $g(t) = \sec(t)$.

$u_1' = \sec(t) \Rightarrow u_1 = \int \sec(t)\,dt \Rightarrow \boxed{u_1 = \ln(|\sec(t) + \tan(t)|) + C_1}$

$u_2' = -\cos(t)\cdot\sec(t) \Rightarrow u_2 = \int -1\,dt \Rightarrow \boxed{u_2 = -t + C_2}$

$u_3' = -\sin(t)\cdot\sec(t) \Rightarrow u_3 = \int -\tan(t)\,dt \Rightarrow \boxed{u_3 = \ln(|\cos t|) + C_3}$

the solution is:

$y = [\ln(|\sec t + \tan t|) + C_1]\cdot 1 + [-t + C_2]\cdot\cos(t) + [\ln(|\cos t|) + C_3]\sin(t)$

$$\boxed{y = \underbrace{\left(C_1 + C_2\cos(t) + C_3\sin(t)\right)}_{\text{homogeneous}} + \underbrace{\ln(|\sec(t) + \tan(t)|) - t\cos(t) + \ln(|\cos(t)|)\sin(t)}_{\text{particular}}}$$