PRACTICE PROBLEMS FOR THE FINAL

(1) Determine the general solution of the given differential equations.

$$y^{(4)} - y = \frac{3}{t}$$
$$y''' - y' = 3\sin t$$

(2) Find the solution of the given initial value problem. Describe how the solution behaves as $x \to 0$.

$$4x^2y'' + 8xy' + 17y = 0, \qquad y(1) = 2, \quad y'(1) = -3$$

(3) Find a power series solution of the given O.D.E.s about the given point x_0; find a recurrence relation for the coefficients of the power series. If possible find the general term of each solution.

$$(4 - x^2)y'' + 2y = 0, \qquad x_0 = 0$$
$$y'' - xy' - y = 0, \qquad x_0 = 1$$

(4) Determine a suitable form of $y(t)$ if the method of undetermined coefficients is to be used.

$$y''' - 3y'' + 2y' = t + e^t$$
$$y'' - y' = 2e^t \sin t$$

(5) Determine the radius of convergence of the given power series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}$$

(6) Use the method of Variation of Parameters to find the general solution of the given I.V.P.s

$$2y'' - 3y' + y = t - 1, \qquad y(0) = y'(0) = 1$$
$$y''' + y' = \sec t, \qquad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = -1$$

(7) Find a power series solution about $x_0 = 1$ of the given differential equation.

$$(x^2 - 2x)y'' + xy' - y = 0$$

(8) Find a lower bound on the radius of convergence of the series solution about x_0 of the given differential equations.

$$y'' + 4y' + 6xy = 0, \qquad x_0 = 0, \quad x_0 = 4$$
$$(1 - x^2)y'' + 4xy' + y = 0, \qquad x_0 = 2, \quad x_0 = 5$$

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(9) Seek a power series solution of the given IVPs about the given point x_0. Find the recurrence relation and write the first four terms of each of the two solutions y_1 and y_2.

$$(1-x)y^{''} + y = 0, \qquad x_0 = 0, \quad y(0) = 1, y^{'}(0) = 5$$

$$2y^{''} + (x+1)y^{'} + 3y = 0, \qquad x_0 = 2, \quad y(2) = -1/2, y^{'}(2) = 7$$