Homework - MS403

Due Tuesday, October 29, 2013

Remember to write on only one side of the sheet.

1. Compute $|Gl_n(\mathbf{F}_p)|$.

2. Let p be prime and let R_p denote the following set of matrices:

$$R_p = \{\begin{pmatrix} a & -b \\ b & a \end{pmatrix} |a, b \in \mathbf{F}_p\}$$

(a) Prove that R_p is a commutative ring.

(b) Prove that R_3 and R_7 are fields, but R_5 is not. Try to determine for which primes p the ring R_p is a field.

3. Let V be an F-vector space and let W be a subspace. Prove there is a one-to-one correspondence between the subspaces of V/W and the subspaces of V that contain W.

4. Let V be an n-dimensional vector space over a field F. Let A_m denote the set of multilinear alternating functions on $V^m = V \times V \times \cdots \times V$ (m times). Note that A_m is a vector space over F.

(a) Prove that if $m > n$, then $A_m = 0$.
(b) Prove that if $m \leq n$, then the dimension of A_m is $\binom{n}{m}$.

5. Each of the following is a basis of $\mathbf{F}_7^3$ over the field $\mathbf{F}_7$.

$$B = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

(a) Find the change of basis matrix from B to C, that is, find the matrix P such that $P[v]_B = [v]_C$ for all $v \in \mathbf{F}_7^3$.

(b) Let A be the following matrix over $\mathbf{F}_7$:

$$\begin{pmatrix} 1 & 0 & -1 \\ 2 & -2 & 0 \\ 3 & 1 & 1 \end{pmatrix}$$

Find the matrix of L_A with respect to the basis B.

(c) Use your answer to (a) to find the matrix of L_A with respect to the basis C.