Final Examination - MS403

December 19, 2013

(20)1. Complete the following definitions:

(a) Let F be a field and V a finite dimensional vector space over F. Let $T : V \to V$ be a linear transformation. An element $\alpha \in F$ is called an <u>eigenvalue</u> for T if

(b) A set F with two binary operations $+$ and $\cdot$ is called a <u>field</u> if

(c) A permutation θ in S_n is called <u>even</u> if

(d) Let G be a group. A set S is said to be a <u>G-set</u> if

(15)2. Give an example of each of the following. No justification is required.

(a) A 2×2 matrix over $\mathbf{Q}$ that has no eigenvalues in $\mathbf{Q}$.

(b) A nonabelian group of order 54.

(c) Two groups G_1 and G_2 and a nontrivial homomorphism $f : G_1 \to G_2$ such that f is not one-to-one and not onto. (The trivial homomorphism is the one that sends every element in G_1 to the identity.)

(10)3.(a) Find the order of the group $Gl_3(\mathbf{F}_5)$.

(b) Find the number of elements of order 6 in S_7.

(10)4.(a) Let F be a field and V a finite dimensional vector space over F. Complete the following definition: A function $V \times V \times \cdots \times V$ (n times) is called multilinear, alternating if

(b) What is the characterization of the determinant function $det : M_n(F) \to F$ using multilinear, alternating functions? No justification required.

(10)5. State the theorem classifying the finite subgroups of the group M_2 of rigid motions of the plane. Your statement should include a description of how these groups are acting on the plane.

(10)6. Find the Sylow-3 subgroups of S_6.

(10)7. Prove the following is a presentation of S_3:

$$< x, y | x^2 = y^2 = (xy)^3 = e >$$

(10)8. Prove there is no simple group of order 150.

(10)9. Let G be a nonabelian group of order p^3, where p is prime.

(a) Prove that $|Z(G)| = p$.

(b) Prove that $G/Z(G)$ is isomorphic to $C_p \times C_p$.