Midterm-MS403

(20)1. Complete the following definitions:

(a) Let G be a group and $g \in G$. The <u>order</u> of g is

(b) If G is a group the <u>center</u> of G is

(c) Let G be a group and S a nonempty subset of G. The subgroup <u>generated</u> by S is

(d) Let G be a group and let x and y be elements of G. We say x and y are <u>conjugate</u> if

(20)2. Give examples of each of the following. No justification is required.

(a) An infinite group in which every element has finite order.

(b) A nonidentity automorphism of S_3.

(c) A subgroup of S_4 that is isomorphic to D_4.

(d) A noncyclic group G with exactly five subgroups (including G and $\{e\}$).

(10)3. Let $S = \mathbf{R}^2 - (0,0)$, the plane with the origin removed. Define a relation $\sim$ on S by $(a,b) \sim (c,d)$ if $ad = bc$.

(a) Prove this is an equivalence relation.

(b) Draw the equivalence classes.

(9)4. Let $G = <g>$ be a cyclic group of order 12. Write down all of the subgroups of G.

(9)5. True or False. No justification required.

(a) Let G and K be finite groups and let $f : G \to K$ be a group homomorphism. For all $x \in G$, the order of $f(x)$ divides the order of x.

(b) If S is a nonempty set with an associative binary operation for which there is an identity element, then the identity is unique.

(c) In an abelian group every sugroup is normal.

(10)6. Let G be a group. Call a subgroup H of G <u>proper</u> if $H \neq G$. Prove that if G is not the union of its proper subgroups, then G is cyclic.

(10)7. (a) Find the largest possible order of an element of S_7.

(b) Find the number of elements in S_6 that are conjugate to the following element:

e ✓

(1246) ✗ ✓

$g_1 \in g_1N$ (35) ✓

$g_2 \in g_1N$ $(1246)(35)$ ✓

$C_x = \{gx g^{-1} \mid g \in S_6\}$

$G(1246)(35)G^{-1}$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 5 & 6 & 3 & 1 \end{pmatrix}$$

$$|C_x| = \frac{|G|}{|C_G(x)|}$$

$\varphi \circ \pi(g_1)$

$\varphi\pi(g_1)$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

$30 \cdot 8 \cdot 3$

90

720

(12)8. Let G_1 and G_2 be groups and let $f : G_1 \to G_2$ be a homomorphism. Let N denote the kernel of f.

(a) Let $\pi : G_1 \to G_1/N$ denote the canonical homomorphism. Describe π explicitly. $\pi(g) = gN$

(b) Find a homomorphism $\varphi : G_1/N \to G_2$ that satisfies the equation $\varphi \circ \pi = f$. Be sure to show your homomorphism is well defined. $f(g_1N) = f(g_2)N$. $f(g_1) = f(g_2)$.

$\varphi \circ \pi(g_1N) = \varphi \circ \pi(g_2N)$ $\varphi(g_1) = \varphi(g_2)$

(1)(a) Let G be a group and $g \in G$. the order of g is the smallest positive integer k s.t. $g^k = e$. If no such integer exists, then the order of g is infinite.

(b) If G is a group, the center of G is $Z(G) = \{g \in G : gx = xg \ \forall x\}$ all elements in G that commute with every other element.

(c) Let G be a group and S a nonempty subset of G. the subgroup generated by S is $\langle S \rangle = \{s_1^{\pm 1} s_2^{\pm 1} \cdots s_r^{\pm 1} \mid r > 0 \text{ and } s_i \in S\}$.

$\langle S \rangle = \bigcap_{\substack{S \subseteq H \\ H \leq G}} H$; smallest subgroup of G that contains S.

(d) Let G be a group and let x an y be elements of G. We say x and y are conjugate if there exists $z \in G$ $zxz^{-1} = y$.

(2)(a) $(\mathbb{Q}, +)/(\mathbb{Z}, +)$.

(b) $f : S_3 \to S_3$; s.t. f is an isomorphism $f(6) = (12)6(12)$

(c) the subgroup $H = \{e, (13)(24), (23), (14), (12)(34), (1243), (14)(23), (1342)\}$

$H \cong D_4$.

(d) ?

$(1246)(1246)(35)(1246)$

$(21)(46)[(1246)(35)](21)(46)$

$(1246)(1246) = [(62)(14)]$

$gx = xg$

$x = gxg^{-1}$