MS403 Review Sheet

Here are topics I have covered in my lectures during the semester. You should know the definitions of all the terms and examples and counterexamples, but as I said in class the emphasis on the examination will be on the second half of the course.

binary operation.
associativity.
commutativity.
identity element.
inverses.
group.
subgroup.
explicit examples of groups, including dihedral groups, cyclic groups, symmetric groups, orthogonal groups, general and special linear groups,.....
relations.
equivalence relations.
cosets.
Lagrange's theorem.
order of an element.
group homomorphisms.
centralizer.
center.
operations on subgroups.
subgroup generated by a set of elements in a group.
normal subgroups.
quotient groups.
fundamental theorem on group homomorphisms.
direct products.
simple groups.
properties of the symmetric group, including cycle decomposition.
conjugacy classes, the class formula.
existence of elements of prime order.
properites of p-groups.
$Aut(G)$, $Inn(G)$.
fields, finite fields.
vector spaces, subspaces.
linear dependence, spanning, basis, dimension.
linear transformations and matrices.
connection to linear systems.
quotient spaces.
change of basis.

eigenvalues, eigenvectors, characteristic polynomial, diagonalization.
trace.
multilinear, alternating functions and determinants.
dot product and orthogonal matrices.
orthonormal bases, Gram-Schmidt process.
rigid motions in $\mathbf{R}^2$, connection to $O_2(\mathbf{R})$ and $SO_2(\mathbf{R})$.
translations, reflections, ...
finite subgroups of $O_2(\mathbf{R})$ and $\mathbf{M}_2$, the group of rigid motions of the plane.
$SO_3(\mathbf{R})$, rotations in $\mathbf{R}^3$.
free group on a set.
presentation of a group.
group actions on a set S, orbits of an action, formula for number of elements in an orbit.
Sylow Theorems (first two), applications to structure of groups.