— M436 — Homework Assignment 10 —

Due: Wednesday, December 3, in class. Each problem is worth 20 points. Please show all your work. Homework that is illegible or discourages the reader otherwise from looking at it will be returned ungraded.

Exercise 1

Find the center and radius of the circle through the points $p_1 = (2/5, 1/5)$ and $p_2 = (3/5, -2/5)$ that is perpendicular to the unit circle.

Exercise 2

Find a hyperbolic isometry of the upper half plane that fixes $(0, 1)$ and that rotates every geodesic through $(0, 1)$ into a geodesic perpendicular to the original one. Write the isometry as a Mobius transformation.

Exercise 3

Given two circles in the plane that have no point in common, show that there is Mobius transformation that takes the two circles to concentric circles.

Exercise 4

Given two circles one inside the other. Form a chain of circles that touch consecutively and always the inner and outer circle. Show that if this chain closes, it closes for all choices of initial circles. See **Figure 1** for an example of chains with seven circles.

Exercise 5

Start by choosing three mutually touching spheres, like the blue, rose, and translucent one in **Figure 2**. The translucent one is touching the others externally. Begin a chain of spheres with a sphere touching all three spheres, like the yellow sphere. Then consecutively add spheres (purple, green, pink, dark blue, moldy green) to the chain so that they touch all three initial spheres and the previously added sphere. Show that the chain of spheres always closes after six spheres.

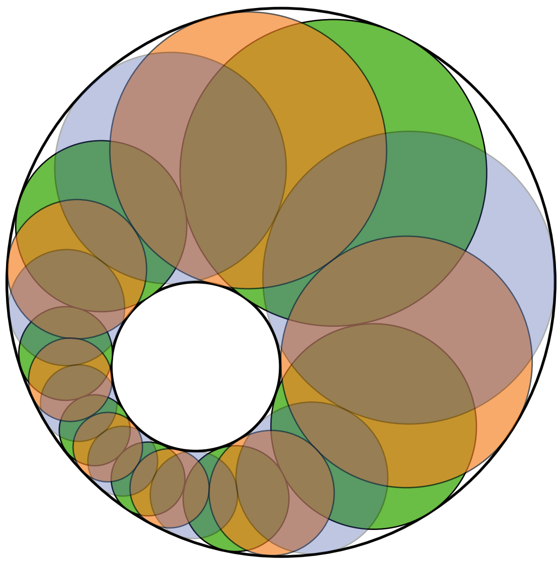

Figure 1 A chain of touching circles

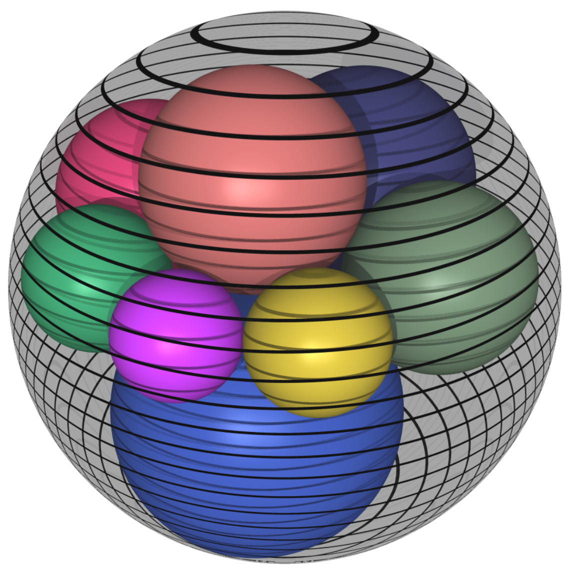

Figure 2 A chain of touching spheres