— M436 — Homework Assignment 4 —

Due:Friday, September 26, in class. Each problem is worth 20 points. Please show all your work.

Exercise 1

Let $D_1 v = 2\left(v - \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ be a dilation in $\mathbf{R}^2$. Find another dilation $D_2 v = \lambda(v - p) + p$ such that $(D_2 \circ D_1)v = v + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Exercise 2

The following puzzle is played on the set of points $\mathbf{Z}^2$ with integer coordinates in $\mathbf{R}^2$. The points $p_1 = (0, 0)$, $p_2 = (1, -1)$, and $p_3 = (-2, 1)$ are 'mirrors', and the player has a peg placed on some point. A move consists of jumping with the peg across any of the three mirrors. For instance, if the peg is at the point $(1, 0)$, we can jump to $(-1, 0)$, $(1, -2)$, or $(5, 2)$, depending on the mirror we use. Find a sequence of jumps that takes a peg at position $(1, 0)$ to position $(1, 2)$ that is different from the solution below.

Another formulation of the problem asks to find a word R in R_1, R_2, R_3, that, when interpreted as a composition of the affine transformations $R_i(v) = -(v - p_i) + p_i$, becomes the translation $R(v) = v + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

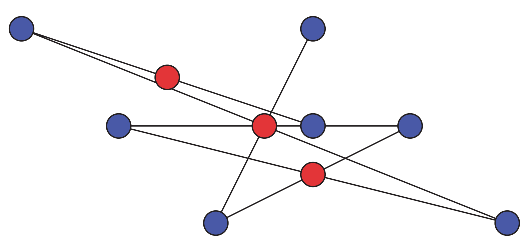

Figure 1 A solution to the jumping puzzle

Exercise 3

Consider the projective plane $\mathbf{F}_3P^2$ over the field with 3 elements. Show that the two triangles with vertices at $p_1 = (1\colon 1\colon 0)$, $p_2 = (1\colon 2\colon 1)$, $p_3 = (0\colon 2\colon 1)$ and $q_1 = (1\colon 0\colon 0)$, $q_2 = (1\colon 1\colon 1)$, $q_3 = (0\colon 0\colon 1)$ are in perspective centrally. Then verify Desargue's theorem by computing the three intersections of corresponding lines (like p_1q_2 with p_2q_1), and showing that they are collinear.

Exercise 4

Show that in the projective plane $\mathbf{F}_3P^2$ over the field with 3 elements, the set of points and lines form a configuration of type 13_4.

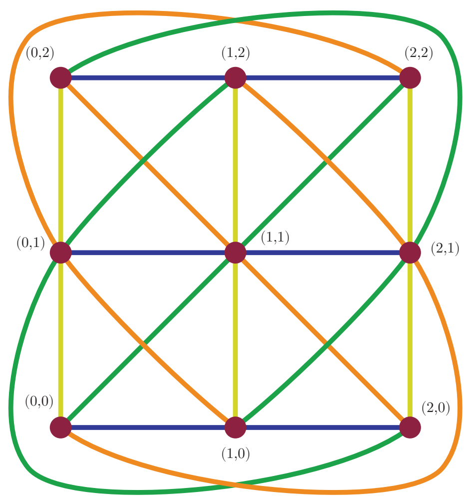

Figure 2 Hesse configuration

Exercise 5

Show that the Hesse configuration can be realized in the complex projective plane $\mathbf{C}P^2$ by writing

$$\begin{aligned} p_{00} &= (0\colon -1\colon 1) & p_{01} &= (-1\colon 0\colon 1) & p_{02} &= (-1\colon 1\colon 0) \\ p_{10} &= (0\colon y\colon 1) & p_{11} &= (x\colon 0\colon 1) & p_{12} &= (y\colon 1\colon 0) \\ p_{20} &= (0\colon x\colon 1) & p_{21} &= (y\colon 0\colon 1) & p_{22} &= (x\colon 1\colon 0) \end{aligned}$$

for suitable complex numbers $x \neq y$.