— M436 — Midterm Preview —

The following 5 problems are multiple choice. Each correct answer is worth 10 points. An incorrect answer results in a 5 point penalty, and no answer is worth 0 points. All problems take place in the real projective plane $\mathbf{RP}^2$. Only one checkmark per problem is allowed. Multiple checkmarks or other ambiguous notation results in 0 points. Do not show any work.

No notes or electronic devices are allowed at any time. The presence of any of these is considered as cheating and will be treated as such.

1. The line through the points $(1:2:1)$ and $(3:1:0)$ is incident with the point

   ☐ $(-2:-1:1)$
   ☒ $(2:-1:-1)$
   ☐ $(2:1:1)$
   ☐ none of these.

2. The lines $2x - 3y + z = 0$ and $x + y + 3z = 0$ are concurrent with the line

   ☐ $x + 5y - 3z = 0$
   ☒ $x - 5y - 3z = 0$
   ☐ $x + 5y + 3z = 0$
   ☐ none of these.

3. The conic $x^2 + 4xy - 2xz + 2yz - z^2 = 0$ has

   ☒ $y - z = 0$
   ☐ $x + y = 0$
   ☐ $x - y = 0$
   ☐ none of these

   as a tangent line.

4. The projective linear transformation given by

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

maps the line $x + 2y + z = 0$ to the line

- ☐ $4x + 3y - 2z = 0$
- ☐ $x - z = 0$
- ☐ $x - y + 3z = 0$
- ☒ none of these.

5. Let T be the projective linear transformation that maps $(0:1:-1)$ to $(1:-1:0)$ to $(-1:0:0)$ to $(0:0:1)$ to $(0:1:-1)$. Then T maps $(1:-1:1)$ to

- ☐ $(1:1:-1)$
- ☒ $(1:-1:1)$
- ☐ $(-1:-1:1)$
- ☐ none of these.

$A \cdot e_i = f_i$

easy: $e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}, e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}, e_4 = \begin{pmatrix}1\\1\\1\end{pmatrix}$

$A = (\lambda_1 f_1 \mid \lambda_2 f_2 \mid \lambda_3 f_3)$

This works because these are the standard basis.

$A e_4 = A(e_1 + e_2 + e_3 + e_4)$

$= \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3$

$= f_4$

In general $A : f_i \to g_i$

$A_1 : e_i \mapsto f_i$

$A_2 : e_i \mapsto g_i$

$A_2 A_1^{-1}$