1) (a) the long term growth rate of this population is $\boxed{\lambda_0 = 0.604504}$ i.e., the population decreases by approximately 60.45% on each generation.

(b) the long Term age distribution of the population is: given by:

$$\begin{bmatrix} -0.985157 \\ -0.162969 \\ -0.0539184 \end{bmatrix} \cdot \frac{1}{-0.985157 - 0.16296 - 0.0539184} = \frac{1}{-1.2020444} \begin{bmatrix} -0.985157 \\ -0.162969 \\ -0.0539184 \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{bmatrix} 0.8195679 \\ 0.13557652 \\ 0.04485558 \end{bmatrix}}$$

this is just the eigenvector associated with the largest eigenvalue normalized to make it a distribution.

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(c) $A = \begin{bmatrix} 0 & 2a & 5a \\ 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix}$: we want to compute $\det(A - \lambda I) = 0$,

$$\Rightarrow 0 = \det(A - \lambda I) = \det\left(\begin{bmatrix} -\lambda & 2a & 5a \\ 0.1 & -\lambda & 0 \\ 0 & 0.2 & -\lambda \end{bmatrix}\right)$$

$$= (-\lambda)\begin{vmatrix} -\lambda & 0 \\ 0.2 & -\lambda \end{vmatrix} - 2a\begin{vmatrix} 0.1 & 0 \\ 0 & -\lambda \end{vmatrix} + 5a\begin{vmatrix} 0.1 & -\lambda \\ 0 & 0.2 \end{vmatrix}$$

$= -\lambda^3 + 0.2a\lambda + 0.1a$. But, we want $\lambda = 1$, so that we solve for: $-1 + 0.2a + 0.1a = 0 \Rightarrow 0.3a = 1 \Rightarrow a = \frac{1}{0.3}$

$\Rightarrow \boxed{a = \frac{10}{3}}$ this is the minimum value of a, so that the population survives in the long Term, i.e., $\lambda = 1$.

2) I would change $\boxed{f_2}$, i.e., the fecundity rate for the second age cohort. this would produce more offspring, wich in turn would have a chance of surviving and producing yet more offspring. Changing f_3 wouldn't have a bigger effect because

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there is a lower probability $[0.1^2]$ of surviving to that age. Finally, changing the survival rates by 20% would imply only a slight increase from 0.1 to 0.12 or 12%, which won't have a bigger effect as that of changing f_2 from 10 to 12.

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3) During busy part of day: c = customers ; h = hours , m = minutes.

$\lambda = 20 \frac{c}{h}$; $\frac{1}{\mu} = 2 \frac{min}{c} \Rightarrow \mu = \frac{1}{2}\frac{c}{m} \cdot \frac{60 m}{1 hr} \Rightarrow \mu = 30 \frac{c}{h}$

NOTE: $\mu > \lambda \Rightarrow$ queue does not explode

(a) the clerk is idle if and only if there is no one in the queuing system. the probability of this happening is:

$$P_0 = \left(\frac{\lambda}{\mu}\right)^0\left(1-\frac{\lambda}{\mu}\right) = 1\cdot\left(1-\frac{20}{30}\right) = 1-\frac{2}{3} = \boxed{\frac{1}{3}}$$

(b) this is the same as the probability of having no customers or only one customer, i.e.,

$$P_0 + P_1 = \frac{1}{3} + \left(\frac{\lambda}{\mu}\right)^1\left(1-\frac{\lambda}{\mu}\right) = \frac{1}{3} + \frac{20}{30}\left(1-\frac{20}{30}\right) = \frac{1}{3} + \frac{2}{3}\left(\frac{1}{3}\right) =$$

$$= \frac{1}{3}\left(1+\frac{2}{3}\right) = \frac{1}{3}\left(\frac{5}{3}\right) = \boxed{\frac{5}{9}}$$

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(c) this is the same as the average total time T in line given by:

$$T = \frac{\lambda}{\mu}\left(\frac{1}{\mu-\lambda}\right) = \frac{20}{30}\left(\frac{1}{30-20}\right) = \frac{\cancel{2}}{3}\frac{1}{\cancel{10}_5} = \boxed{\frac{1}{15}\frac{hr}{c}}$$

, So, on average, each customer waits $\boxed{\frac{1}{15} \text{ of an hour} = 4 \text{ mins}}$ in line waiting for the clerk.

(d) $\lambda = 20 \frac{c}{hr} \xrightarrow{+10\%} \lambda = 22 \frac{c}{hr}$, we want to solve for μ:

$$T = \frac{1}{15} = \frac{22}{\mu}\left(\frac{1}{\mu-22}\right) \Rightarrow \frac{1}{15} = \frac{22}{\mu(\mu-22)} \Rightarrow \mu(\mu-22) = 15\times 22$$

$$\Rightarrow \mu^2 - 22\mu - 330 = 0 \Rightarrow \mu = \left(22 \pm \sqrt{484+1320}\right)/2 \Rightarrow \mu = \left(22\pm\sqrt{1804}\right)/2$$

$\Rightarrow$ Since μ must be a positive number, the clerk would have to work at a rate of $\mu = (22+\sqrt{1804})/2$, in order to keep the average waiting time the same.

So $\mu \simeq 32.236$, meaning the clerk has to increase efficiency by $32.236 - 30 = \boxed{2.236}$

$\Rightarrow$ About a little more than 2 customers per hour. $\boxed{\text{like } 2\frac{1}{4}}$