Enrique Areyan

Exam 2 2014 M447

In your answers you may use the letters P, N, Q, R, and A to refer to the usual matrices and lower-case letters with subscripts, e.g. $n_{1,3}$ to refer to entries in those matrices.

1. A board game championship playoff between two players has the following format. They play one game after another, with the winner and loser of each game recorded (there are no draws). The winner is the first player to have won three more games than the opponent. Assume the players are evenly matched, so that the probability of each player winning any particular game is 50%.

   (a) Formulate this playoff as a Markov chain by giving the transition diagram.

   (b) Now write down the transition matrix of the M.C. in canonical form.

   (c) What are the matrices R and Q?

   (d) What is the expected number of games that must be played to complete this playoff? For this and the following two questions just give your anwers as formulas using the usual matrices.

   (e) What is the expected number of times that player 1 will be one point away from winning the tie-breaker?

   (f) If player 1 wins the first game, what is the probability that she wins the playoff?

2. [3 pt.s each] For each of the following transition matrices, say whether the corresponding M.C. is ergodic, regular, or absorbing.

(a)
$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(b)
$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \end{array}$$

(c)
$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

3. [6 pt.s each] Shown is a diagram of a fluid divided into three regions. The number in each region represents the pressure there. A particle is observed at regular time intervals as it flows at random from region to region (possibly staying in the same region). Assume that the probabilities of each possible transition *inversely* proportional to the pressure in the new region.

4 A	3 B	2 C

   (a) Formulate this as a Markov chain, and give the transition matrix.

   (b) Is this M.C. ergodic?

   (c) Calculate the vector W of stable probabilities.

   (d) In the long run what fraction of the time would the particle spend in the middle region?