1. [12 pt.s] A population model with four age cohorts has the growth matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & .2 & 0 \end{bmatrix}$$

   (a) What is the long term growth rate of this population?

   (b) What is the long term age distribution of the population?

   (c) Suppose the fecundity rates for all age cohorts are changed by a factor of a, where a is a real constant. What is the minimum value of a that would allow the population to survive in the long term?

   (d) Suppose the survival rates for all age cohorts are changed by a factor of b, where b is a real constant. What is the minimum value of b that would allow the population to survive in the long term?

2. [6 pt.s] Suppose another population with four age cohorts has growth matrix

$$A = \begin{bmatrix} 1 & 10 & 20 & 5 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \end{bmatrix}.$$

   If you could increase one of the nonzero values in that matrix by 20%, i.e. multiply it by 1.2, which one would you choose to change in order to maximize the growth rate?

3. [12 pt.s] Suppose a small local bar has only one ID checker working at a time. During the busy part of the night they expect an average of 100 people per hour to attempt to enter, while the checker requires an everage of 10 seconds to check each ID.

   (a) What is the probability that the checker is idle at a random moment?

   (b) What is the probability that there is no entrance line?

   (c) How much time on average does a person spend waiting outside the bar to get in?

   (d) If the average number of people attempting to enter the bar per hour increased by 20%, by how much would the checker need to increase his efficiency so that the average time spent waiting outside remains the same?

   (e) If the checker increased his efficiency by 20%, by how much could the popularity of the bar increase without the average time waiting outside increasing from the original value?