Practice Exam 2 **2014** **M447**

1. A tie-breaker in tennis is played as follows. Player 1 serves for the first point, and thereafter each player, player 2 serves for the next two points, and thereafter the players alternate serving for two consecutive points. The player who serves has a 60% chance of winning the point. The first player to score at least six points while also having at least two more points than the opponent wins the tie-breaker.

   (a) Suppose the score in the tie-breaker is currently 3-3. Formulate the remainder of the tie-breaker as a Markov chain by giving the transition diagram.

   (b) Now write down the transition matrix of the M.C. from part 1a in canonical form.

   (c) What are the matrices R and Q?

   (d) What is the expected number of future points that must be played to complete this tie-breaker?

   (e) What is the expected number of times that player 1 will be one point away from winning the tie-breaker?

   (f) Player 2 serves for the next point. If player 2 wins that point, what will be the probability that player 2 also wins the tie-breaker?

2. For each of the following transition matrices, say whether the corresponding M.C. is ergodic, regular, and/or absorbing.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

3. Shown is the floorplan of a mouse den with four rooms. Suppose that the mouse always passes through doors at random, choosing from the doors accessible from its present room with equal probability. In the long run which doors would it pass through most frequently?