# M451/551 Quiz 4

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You do not need to simplify numerical expressions.

1. Suppose you pay 5 to buy a European ($K = 100$, $t = 1/2$) put option on a given security. Assuming a nominal annual interest rate of 6 percent, compounded monthly, find the present value of your net return from this investment if

   (a) $S(1/2) = 102$;

   (b) $S(1/2) = 98$.

$$\text{value of put option at time } t = \begin{cases} K - S(t) & \text{if } S(t) < K \\ 0 & \text{if } S(t) \geq K \end{cases}$$

Hence, (a) value $= 0$ since $S(1/2) = 102 \geq 100 = K$

So the p.v. of net return is $\boxed{-5\ \$}$

(b) value $= 2$ since $S(1/2) = 98 < 100 = K$

So the p.v. of this option is: $2\left(1 + \frac{0.06}{12}\right)^{-6} = 2(1.005)^{-6}$.

Hence, the p.v. of net return is:

$$\boxed{\underbrace{2(1.005)^{-6}}_{\text{p.v. of option}} - \underbrace{5}_{\text{cost of option}}}$$ in p.v. dollars.

+10

(Problem #2 is on the other side.)

2. At t=0, the price of a certain stock is S(0)=\$50. At $t = 1$, the price is either S(1)=\$80 or S(1)=\$30. A certain option contract is worth \$10 if the stock price is \$80, and is worth \$0 if the stock price is \$30. Assuming no arbitrage opportunities, and continuously compounded interest of 5%, what is the price of the option at time t=0?

Let $x$ be # of shares and $y$ # of options.

then value of portfolio @ time 1 is

$$\begin{cases} 80x + 10y & \text{if } S(1) = 80 \\ 30x & \text{if } S(1) = 30. \end{cases}$$

we want to have: $80x + 10y = 30x \Rightarrow 50x = -10y \Rightarrow y = -5x$

To price the option consider: gain = value − cost, where

cost = $50x + Cy = 50x - 5xC$. @ time 0

value = $30x$ @ time 1, so value = $30xe^{-0.05}$ @ time 0.

Hence gain = $30xe^{-0.05} - 50x + 5xC$ @ time 0

But we want no arbitrage, which means gain = 0 $\Rightarrow$

$$0 = 30xe^{-0.05} - 50x + 5xC$$
$$= x(30e^{-0.05} - 50 + 5C),$$ since we assume $x > 0$, it follows:

$$30e^{-0.05} - 50 + 5C = 0 \Rightarrow 5C = 50 - 30e^{-0.05}$$
$$\Rightarrow \boxed{C = 10 - 6e^{-0.05}}$$

So the cost of the option, assuming no arbitrage, in present value dollars is $\boxed{\$10 - 6e^{-0.05}}$

+10