

M451/551 Quiz 5
February 17, Prof. Connell

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You do not need to simplify numerical expressions.

1. In Example 6.1b, a stock with $S(0) = 100$ after one year is either $S(1) = 200$ or $S(1) = 50$ and that the risk-neutral probability of $S(1) = 200$ is $p = \frac{1+2r}{3}$. Suppose one also has the option of purchasing a put option that allows its holder to put the stock for sale at the end of one period for a price of 150. Determine the value of P , the cost of the put, if there is to be no arbitrage and assuming simple compounding at interest rate r .

$$100 \begin{cases} 200 & p = \frac{1+2r}{3} \\ 50 & 1-p = 1 - \frac{1+2r}{3} = \frac{2-2r}{3} \end{cases}$$

To determine the value of P , let us see what the value of the put option is

$$\text{P.V. return on put option} = \begin{cases} -P & \text{if } S(1) = 200 \\ 100(1+r)^{-1} - P & \text{if } S(1) = 50 \end{cases}$$

$$\begin{aligned} E[\text{P.V. return on put}] &= p \cdot (-P) + (1-p) [100(1+r)^{-1} - P] = \\ &= -pP + 100(1+r)^{-1} - P - p100(1+r)^{-1} + pP \\ &= 100(1+r)^{-1}(1-p) - P \\ &= 0 \quad (\text{for no arbitrage}) \end{aligned}$$

$$\Rightarrow P = 100(1+r)^{-1}(1-p)$$

$$\Rightarrow P = \frac{100(2-2r)}{3(1+r)}$$

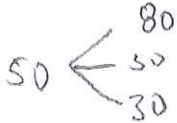
$$\Rightarrow \boxed{P = \frac{200(1-r)}{3(1+r)}}$$

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(Problem #2 is on the other side.)

2. Now let's do three branches, and again assume a nominal yearly interest rate of 10% compounded every period of 1 year. Suppose $S(0) = 50$ and that $S(1)$ is one of 80, 50, or 30. In the setting of the arbitrage theorem, we have $n = 1$, $m = 3$ and the profit return matrix is: $R = (30, 0, -20)$.

- (a) Find a single vector $(p_1, 1/2, p_3)$ (i.e. with $p_2 = 1/2$) that explains the stock price under no-arbitrage. (There is one.)
 (b) The call option with strike $K = 60$ at time 1 has price C . If you rely on the probability vector you found in part a, what is C ?



$$(a) R \cdot (p_1, 1/2, p_3) = 0 \Rightarrow 30p_1 + 0 \cdot 1/2 - 20p_3 = 0$$

$$\text{AND } p_1 + p_2 + p_3 = 1 \Rightarrow 30p_1 = 20p_3$$

$$\text{AND } p_i \geq 0 \quad \forall i=1,2,3 \Rightarrow p_1 = \frac{2}{3}p_3$$

$$\text{So, } p_1 + p_2 + p_3 = 1 \Rightarrow \frac{2}{3}p_3 + \frac{1}{2} + p_3 = 1 \Rightarrow p_3 \left(\frac{2}{3} + 1 \right) = \frac{1}{2}$$

$$\Rightarrow p_3 = \frac{3}{10} \quad \text{AND } p_1 = \frac{2}{3}p_3 = \frac{2}{3} \cdot \frac{3}{10} \Rightarrow p_1 = \frac{2}{10}$$

The vector is $\left(\frac{2}{10}, \frac{1}{2}, \frac{3}{10} \right)$

(b) Present value call option = $\begin{cases} 20(1.1)^{-1} - C & \text{if } S(1) = 80 \\ \cancel{10(1.1)^{-1} - C} & \text{if } S(1) = 50 \\ -C & \text{if } S(1) = 30 \end{cases}$ with $\left. \begin{matrix} p_1 = \frac{2}{10} \\ p_2 = \frac{1}{2} \\ p_3 = \frac{3}{10} \end{matrix} \right\}$

$$E[\text{P.V. return}] = p_1 \cdot [20(1.1)^{-1} - C] + p_2 [10(1.1)^{-1} - C] + p_3 (-C)$$

$$= -C(p_1 + p_2 + p_3) + 20(1.1)^{-1}p_1 + 10(1.1)^{-1}p_2$$

$$= -C + 4(1.1)^{-1} + 5(1.1)^{-1} \quad \dots \text{ since } p_1 + p_2 + p_3 = 1 \text{ AND}$$

$$= 0 \quad (\text{no arbitrage})$$

$$\Rightarrow C = 9(1.1)^{-1}$$

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