M451/551 Quiz 5

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You do not need to simplify numerical expressions.

1. In Example 6.1b, a stock with $S(0) = 100$ after one year is either $S(1) = 200$ or $S(1) = 50$ and that the risk-neutral probability of $S(1) = 200$ is $p = \frac{1+2r}{3}$. Suppose one also has the option of purchasing a put option that allows its holder to put the stock for sale at the end of one period for a price of $\boxed{150}$. Determine the value of P, the cost of the put, if there is to be no arbitrage and assuming simple compounding st interest rate r.

$$100 \begin{cases} 200 \\ 50 \end{cases} \qquad p = \frac{1+2r}{3} \qquad 1-p = 1 - \frac{1+2r}{3} = \frac{2-2r}{3}$$

To determine the value of P, let us see what the value of the put optia is

$$\begin{array}{l} \text{P.V} \\ \text{return} \\ \text{on put} \\ \text{option} \end{array} = \begin{cases} -P & \text{if } S(1) = 200 \\ 100(1+r)^{-1} - P & \text{if } S(1) = 50 \end{cases}$$

$$E\left[\begin{array}{l} \text{P.V.} \\ \text{return} \\ \text{on put} \end{array}\right] = p\cdot(-P) + (1-p)\left[100(1+r)^{-1} - P\right]$$

$$= \cancel{-pP} + 100(1+r)^{-1} - P - p100(1+r)^{-1} + \cancel{pP}$$

$$= 100(1+r)^{-1}(1-p) - P$$

$$= 0 \quad (\text{for no arbitrage})$$

$$\Rightarrow P = 100(1+r)^{-1}(1-p)$$

$$\Rightarrow P = \frac{100(2-2r)}{3(1+r)}$$

$$\Rightarrow \boxed{P = \frac{200(1-r)}{3(1+r)}}$$

+10

(Problem #2 is on the other side.)

2. Now let's do three branches, and again assume a nominal yearly interest rate of 10% compounded every period of 1 year. Suppose $S(0) = 50$ and that $S(1)$ is one of 80, 50, or 30. In the setting of the arbitrage theorem, we have $n = 1$, $m = 3$ and the profit return matrix is: $R = (30, 0, -20)$.

   (a) Find a single vector $(p_1, 1/2, p_3)$ (i.e. with $p_2 = 1/2$) that explains the stock price under no-arbitrage. (There is one.)

   (b) The call option with strike $K = 60$ at time 1 has price C. If you rely on the probability vector you found in part a, what is C?

$$50 \left\langle \begin{matrix} 80 \\ 50 \\ 30 \end{matrix} \right.$$

(a) $R \cdot (p_1, 1/2, p_3) = 0 \Rightarrow 30p_1 + 0 \cdot 1/2 - 20p_3 = 0$

AND $p_1 + p_2 + p_3 = 1$ $\Rightarrow 30p_1 = 20p_3$

AND $p_i \geq 0 \ \forall i = 1,2,3$ $\Rightarrow p_1 = \frac{2}{3}p_3$

So, $p_1 + p_2 + p_3 = 1 \Rightarrow \frac{2}{3}p_3 + \frac{1}{2} + p_3 = 1 \Rightarrow p_3\left(\frac{2}{3} + 1\right) = \frac{1}{2}$

$\Rightarrow \boxed{p_3 = \frac{3}{10}}$ AND $p_1 = \frac{2}{3}p_3 = \frac{2}{\cancel{3}}\frac{\cancel{3}}{10} \Rightarrow \boxed{p_1 = \frac{2}{10}}$

The vector is $\boxed{\left(\frac{2}{10}, \frac{1}{2}, \frac{3}{10}\right)}$

(b). Present value call option $= \begin{cases} 20(1.1)^{-1} - C & \text{if } S(1) = 80 \\ \cancel{10(1.1)^{-1}} - C & \text{if } S(1) = 50 \\ -C & \text{if } S(1) = 30 \end{cases}$ wit $p_1 = \frac{2}{10}$, $p_2 = \frac{1}{2}$, $p_3 = \frac{3}{10}$

$$E\left[\begin{matrix}\text{P.V.} \\ \text{return}\end{matrix}\right] = p_1 \cdot [20(1.1)^{-1} - C] + p_2[10(1.1)^{-1} - C] + p_3(-C)$$

$= -C(p_1 + p_2 + p_3) + 20(1.1)^{-1}p_1 + 10(1.1)^{-1}p_2$ AND

$= -C + 4(1.1)^{-1} + 5(1.1)^{-1}$ ... since $p_1 + p_2 + p_3 = 1$

$= 0$ (no arbitrage)

$\Rightarrow \boxed{C = 9(1.1)^{-1}}$

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