M451/551 Quiz 9
April 7, Prof. Connell

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You do not need to simplify numerical expressions.

1. If β_i is the beta of stock i for $i = 1, ..., k$, what would be the beta of a portfolio in which α_i is the fraction of ones capital that is used to purchase stock i ($i = 1, ..., k$)? (Assume $\sum_{i=1}^{k} \alpha_i = 1$.)

Our rate of return on the portfolio, say R_p, is given by

$$R_p = \sum_{i=1}^{k} \alpha_i R_i \quad \cdots (1)$$

Also, for a given security i $\Rightarrow$ $\beta_i = \dfrac{Cov(R_i, R_m)}{Var(R_m)} \Rightarrow Cov(R_i, R_m) = \beta_i Var(R_m) \cdots (2)$

So, let β_p be the Beta of our portfolio. According to the previous equation:

$$\beta_p = \frac{Cov(R_p, R_m)}{Var(R_m)} = \frac{Cov\left(\sum_{i=1}^{k} \alpha_i R_i, R_m\right)}{Var(R_m)} \quad \text{by (1)}$$

$$= \frac{\sum_{i=1}^{k} \alpha_i Cov(R_i, R_m)}{Var(R_m)} \quad \text{by bilinearity of covariance}$$

$$= \sum_{i=1}^{k} \alpha_i \frac{\beta_i Var(R_M)}{Var(R_M)} \quad \text{by (2)}$$

$$= \frac{Var(R_M)}{Var(R_M)} \sum_{i=1}^{k} \alpha_i \beta_i \quad \text{factoring constants}$$

$$= \sum_{i=1}^{k} \alpha_i \beta_i$$

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(Problem #2 is on the other side.)

So, the beta of our portfolio is just the weighted sum of the betas for each security.

2. Let X_i be a Poisson random variable with mean λ_i. If $\lambda_1 \geq \lambda_2$, show that $X_1 \geq_{lr} X_2$.

Poisson R.V. is a discrete R.V. Hence, we want to show that, if X_1 and X_2 are Poisson with mean λ_1 and λ_2 resp.

Then $\dfrac{P(X_1 = k)}{P(X_2 = k)}$ is increasing in $k \in \{0, 1, 2, \dots\}$ provided $\lambda_1 \geq \lambda_2$.

By definition $P(X_i = k) = \dfrac{\lambda_i^k}{k!} e^{-\lambda_i}$. thus,

$$f(k) = \frac{P(X_1 = k)}{P(X_2 = k)} = \frac{\frac{\lambda_1^k}{k!} \cdot e^{-\lambda_1}}{\frac{\lambda_2^k}{k!} e^{-\lambda_2}} = \frac{\lambda_1^k \cdot \cancel{k!}\, e^{-\lambda_1}}{\lambda_2^k \cdot \cancel{k!}\, e^{-\lambda_2}} = \left(\frac{\lambda_1}{\lambda_2}\right)^k \cdot e^{\lambda_2 - \lambda_1}.$$

So the function $f(k)$ is of the form $f(k) \propto a \cdot b^k$, where $b > 1$ only because $\lambda_1 \geq \lambda_2 \Rightarrow \dfrac{\lambda_1}{\lambda_2} \geq 1$. and $a > 0$ because the function $e^{\lambda_2 - \lambda_1}$ is always positive. Clearly $f(k)$ is an increasing function of $k \in \{0, 1, 2, \dots\}$.

this shows that $X_1 \geq_{lr} X_2$.

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