

M463 Homework 10

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A building has 10 floors above the basement. If 12 people get into an elevator at the basement, and each chooses a floor at random to get out, independently of the others, at how many floors do you expect the elevator to make a stop to let out one or more people of these 12 people?

Solution:

Let Y = number of floors the elevator stops at. Then, $Y = X_1 + X_2 + \cdots + X_{10}$, where:

$$X_i = \begin{cases} 1 & \text{if the elevator stops at the } i\text{th floor} \\ 0 & \text{otherwise} \end{cases}$$

Note that the probability of a single person getting out in any floor is $p = \frac{1}{10}$. The distribution of X_i is:

$$\begin{aligned} P(X_i = 1) &= P(\text{at least one person gets out at floor } i\text{th}) = 1 - P(\text{no one gets out at floor } i\text{th}) \\ &= 1 - \binom{12}{0} \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{12} \\ &= 1 - \left(\frac{9}{10}\right)^{12} \end{aligned}$$

Therefore, $E(X_i) = 1 - \left(\frac{9}{10}\right)^{12}$ for $1 \leq i \leq 10$. Now, by linearity of the expected value:

$$E(Y) = E(X_1 + X_2 + \cdots + X_{10}) = E(X_1) + E(X_2) + \cdots + E(X_{10}) = 10 \times \left[1 - \left(\frac{9}{10}\right)^{12} \right] = \boxed{7.17570463519}$$

Hence, we would expect the elevator to make a stop to let out one or more people approximately at 7.176 floors.