Name: Enrique Areyan **2013 June 28** **M463**

Exam 1

Justify your answers!
Numerical expressions may be left unsimplified.

10 1. [10 pt.s] Of those IU students who are Music Majors, 20% are Lady Gaga fans; of those who are not Music Majors, 40% are fans of Lady Gaga. All together 35% of IU students are Lady G. fans.

(a) What is the overall percentage of IU students who are Music majors?

(b) Suppose a randomly selected IU student happens to be a Lady G. fan. What is the probability s/he is a Music Major?

10 2. [10 pt.s] Suppose that $P(E) = .6$, event F is independent of E, and $P(E \cup F) = .8$. Find $P(F)$.

Hint: Let $x = P(F)$, and solve for x algebraically after applying the inclusion-exclusion formula and the independence formula.

10 3. [10 pt.s] A standard deck of cards has 52 cards, in 13 ranks (2, 3, 4, ..., 10, J, Q, K, A) and four suits (♢, ♡, ♣, ♠). If a hand of five cards is dealt at random, find the probability that it the hand is a full house, meaning it contains three cards of one rank and two cards of another rank.

10 4. [10 pt.s] Suppose a pollster randomly calls 1000 people and asks whether they support Obama's healthcare plan. Suppose that in fact 50% of the general population supports the plan. Let X be the number of "Yes" responses. You may assume that X has a binomial distribution.

(a) Find a 95% confidence interval for X.

(b) How large a sample size n would be needed so that the sample fraction X/n would be within 0.01 of the true value 0.50 (with confidence level 95%)?

10 5. [10 pt.s] Let X be a R.V. with a Poisson($\overset{=\mu}{2}$) distribution. Find

$$P(X = 2 \mid X = 1 \text{ or } X = 2).$$

(2) $P(E) = .6$; $P(E \cup F) = .8$. Since E & F are independent $P(E \cap F) = P(E) \cdot P(F)$.
$P(F) = x$. So, $P(E \cap F) = .6x$.

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$.8 = .6 + x - .6x \Rightarrow .2 = x(1 - .6) \Rightarrow .2 = .4x \Rightarrow x = \frac{.2}{.4} = \frac{\frac{2}{10}}{\frac{4}{10}} = \frac{2}{4} = \frac{1}{2}$

So, $\boxed{P(F) = \frac{1}{2}}$ this works since $P(E \cap F) = P(E) \cdot P(F) = .6 \times .5 = .3$ AND
$P(E \cup F) = P(E) + P(F) - P(E \cap F) = .6 + .5 - .3 = .8$

(3) $P(\text{Full house}) = \frac{\#\text{ full house}}{\# \Omega} = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$ the number of full house hands can be obtained as follow: Pick a rank. there are $\binom{13}{1}$ ways of doing this. then, by multiplication principle there are $\binom{13}{1}\binom{4}{3}$ ways of picking three cards of this rank, 4 because of suits →

Since we already have 3 cards of the same rank, we have to complete with 2 cards of a different rank. Since we already picked a rank, there are $\binom{12}{1}$ ways of picking the remaining rank togheter with $\binom{4}{2}$ ways of picking cards of this last rank. So, all in all, using the multiplication principle we have that the number of full house hands are : $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}$. Now, by E.L.O principle, since we have $\binom{52}{5}$ total number of hands $\Rightarrow P(\text{full house}) = \frac{\#F.H}{\#\Omega} = \boxed{\frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}}$

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(4) $n = 1000$. $p = 1/2$ = true probability of supporting Obama's plan.

LO X = # of yes in the sample. $X \sim \text{Binomial}(n=1000, p=1/2)$.

(a) 95% confidence interval for X, i.e., $\left|X - \frac{1}{2}\right| = 0$. So:

$P\left(\left|X - \frac{1}{2}\right| = 0\right) = .95 \Leftrightarrow P\left(X = \frac{1}{2}\right) = .95$

We are going to use the normal approx to the Binomial with $\mu = n \cdot p = 500$; $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{250}$

So, $P\left(X = \frac{1}{2}\right) = P\left(\frac{X-\mu}{\sigma} = \frac{\frac{1}{2} - 500}{\sqrt{250}}\right) \simeq P\left(Z = \frac{-499.5}{\sqrt{250}}\right)$. Finally, use continuity correction.

$P\left(Z = \frac{-499.5}{\sqrt{250}}\right) = P\left(\frac{-500}{\sqrt{250}} \le Z \le \frac{-499}{\sqrt{250}}\right)$. So $2\Phi(z_0) - 1 = .95 \Rightarrow z_0 = \frac{-500}{\sqrt{250}}$, So the interval is $[\mu - z_0, \mu + z_0]$

(b) We are interested in finding $n = ?$ such that the event $\left|\bar{X} - \frac{1}{2}\right| \le 0.01$ has probability $P\left(\left|\bar{X} - \frac{1}{2}\right| \le 0.01\right) = .95\%$.

Again, using the normal approximation $\mu = n \cdot p = \frac{n}{2}$; $\sigma = \sqrt{npq} = \sqrt{n \frac{1}{2} \frac{1}{2}} = \sqrt{\frac{n}{4}} = \frac{\sqrt{n}}{2}$

$P\left(\left|\bar{X} - \frac{1}{2}\right| \le 0.01\right) = P\left(-0.01 \le \bar{X} - \frac{1}{2} \le 0.01\right) = P\left(-n \cdot 0.01 \le X - \frac{n}{2} \le n \cdot 0.01\right)$

$= P\left(\frac{-n \cdot 0.01}{\sqrt{n}/2} \le \frac{X - \mu}{\sigma} \le \frac{n \cdot 0.01}{\sqrt{n}/2}\right) = .95\% \approx P\left(\frac{-n \cdot 0.01}{\sqrt{n}/2} \le Z \le \frac{n \cdot 0.01}{\sqrt{n}/2}\right)$, where $Z \sim \text{Normal}(\mu=0, \sigma=1)$. So $2\Phi(z_0) - 1 = .95 \Rightarrow$

$z_0 = 2 = \frac{0.01n}{\sqrt{n}/2} = \frac{2 \times 0.01n}{\sqrt{n}} \Rightarrow \sqrt{n} = 0.01n \Rightarrow \boxed{n = 10{,}000}$

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(5) Let $X \sim \text{Poisson}(\mu = 2)$. then $P(X=2 \mid X=1 \text{ or } X=2) = \frac{P(X=2 \cap (X=1 \text{ or } X=2))}{P(X=1 \text{ OR } X=2)}$

$= \frac{P(X=2)}{P(X=1 \text{ OR } X=2)}$; Since $X=2$ is stricter than $X=1$ or $X=2$.

Now, using additive principle since $X=1 \cap X=2 = \phi$.

$= \frac{P(X=2)}{P(X=1) + P(X=2)} = \frac{\frac{e^{-2} \cdot 2^2}{2!}}{\frac{e^{-2} \cdot 2^1}{1!} + \frac{e^{-2} \cdot 2^2}{2!}} = \frac{\frac{4e^{-2}}{2}}{2e^{-2} + \frac{4e^{-2}}{2}} = \frac{2e^{-2}}{2e^{-2} + 2e^{-2}} = \frac{2e^{-2}}{4e^{-2}} = \boxed{\frac{1}{2}}$

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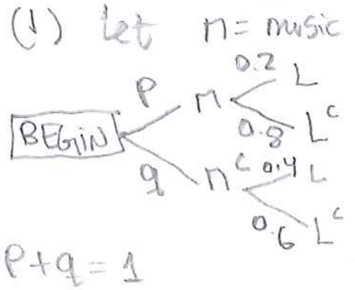
(1) Let M = music major ; L = Lady Gaga fan $\Rightarrow$ Since $M \cup M^c$ is a partition of Ω.



a) Using total prob. $P(L) = P(L \mid M) \cdot P(M) + P(L \mid M^c) \cdot P(M^c)$

$\Rightarrow 0.35 = 0.2p + 0.4(1-p) \Rightarrow 0.35 = 0.2p + 0.4 - 0.4p$

$\Rightarrow 0.35 = 0.4 - 0.2p \Rightarrow 0.2p = 0.4 - 0.35 \Rightarrow 0.2p = 0.05$

$\Rightarrow p = \frac{0.05}{0.2} = \frac{5 \times 10^{-2}}{2 \times 10^{-1}} = \frac{5}{2} 10^{-1} = \frac{5}{20} = \boxed{\frac{1}{4}}$. So $\boxed{P(M) = 0.25}$

$p + q = 1$

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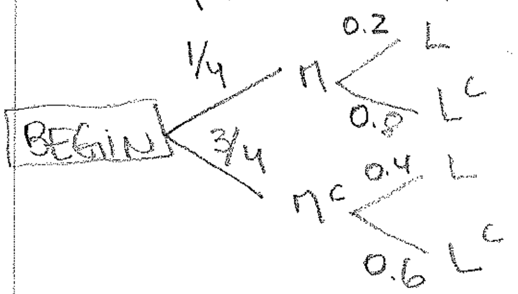
Enrique Areyan M463-Exam 1

Problem 1; part (b):

the updated tree is:



So; $P(M|L) = \dfrac{P(M \cap L)}{P(L)} \Rightarrow P(M \cap L) = P(M|L) \cdot P(L)$

But; $P(L|M) = \dfrac{P(M \cap L)}{P(M)} \Rightarrow P(M \cap L) = P(L|M) \cdot P(M)$

$= 0.2 \times 0.25$

$= \dfrac{\frac{2}{10}}{\frac{25}{100}} = \dfrac{200}{250} = \dfrac{20}{25} = \dfrac{4}{5}$

Hence,

$P(M \cap L) = P(M|L) \cdot P(L) = P(L|M) \cdot P(M)$ (Baye's Rule)

$P(M|L) = \dfrac{P(L|M) \cdot P(M)}{P(L)} = \dfrac{0.2 \times 0.25}{0.35} = \dfrac{\frac{2}{10} \frac{25}{100}}{\frac{35}{100}} = \dfrac{\frac{50}{1000}}{\frac{35}{100}}$

$= \dfrac{5\cancel{000}}{35\cancel{000}} = \dfrac{5}{35} = \boxed{\dfrac{1}{7}}$