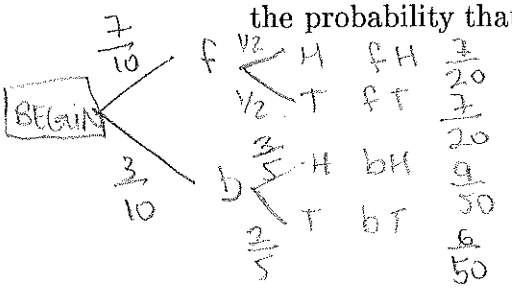
Name: **1999 Summer II** **M463**
Week 1

1. A box contains 4 tickets numbered 1, 2, 3, and 4. Two tickets are drawn from the box at random without replacement. Find the chance that the numbers on the two tickets differ by two or more. $\Omega = \{(x,y) : x,y \in \{1,2,3,4\} \wedge x \neq y\}$. $|\Omega| = 4 \cdot 3 = 12$.

$P(\text{differ by two or more}) = P(\{(1,3),(1,4),(2,4),(3,1),(4,1),(4,2)\}) = \frac{6}{12} = \boxed{\frac{1}{2}}$ ✓

2. A hat contains 10 coins, 7 of which are fair, and 3 of which are biased to land heads with probabililty .6. A coin is chosen from the hat, tossed, and lands heads. What is the probability that it is a fair coin?



$P(\text{fair coin} \mid \text{landed heads}) = \frac{P(fH)}{P(\text{Heads})} = \frac{P(fH)}{P(\{fH, bH\})}$

$= \frac{\frac{7}{20}}{\frac{7}{20} + \frac{9}{50}} = \frac{\frac{7}{20}}{\frac{35+18}{100}} = \frac{700}{20 \cdot 53} = \frac{70 \cdot 10}{2 \cdot 53 \cdot 10} = \frac{35}{106}\ \boxed{\frac{35}{53}}$ ✓

3. Suppose A and B are independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. Find the following:

(a) $P(AB) = \frac{1}{3} \cdot \frac{1}{4} = \boxed{\frac{1}{12}}$ ✓

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{4+3-1}{12} = \frac{6}{12} = \boxed{\frac{1}{2}}$ ✓

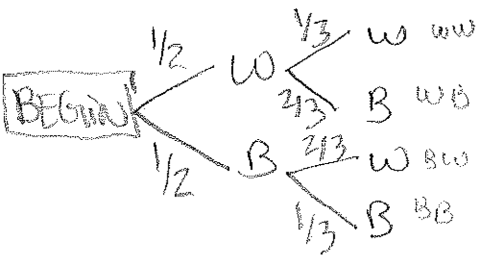
(c) Let C be the event that *exactly* one of A and B occur. Find $P(C)$.

$P(C) = P(AB^c \cup A^cB) = P(AB^c) + P(A^cB) = P(A) - P(AB) + P(B) - P(AB)$

$= P(A) + P(B) - 2P(AB)$

$= \frac{1}{3} + \frac{1}{4} - \frac{2}{12} = \frac{4+3-2}{12} = \boxed{\frac{5}{12}}$ ✓

4. A box contains one black ball and one white ball. A ball is drawn at random, then replaced in the box with an additional ball of the *opposite* color. Then a second ball is drawn from the three balls now in the box. What is the probability that the first ball drawn was white, given that at least one the two balls drawn was *black*?



$P(\text{first } W \mid \text{at least one was black}) = \frac{P(\text{first } W \cap \text{at least } 1b)}{P(\text{at least } 1b)}$

$= \frac{P(WB)}{P(\{WB, BW, BB\})} = \frac{\frac{2}{6}}{\frac{2}{6} + \frac{2}{6} + \frac{1}{6}} = \frac{\frac{2}{6}}{\frac{5}{6}} = \boxed{\frac{2}{5}}$ ✓