Name: **1999 Summer II** **M463**

Quiz 2

1. Cards are dealt from a shuffled, standard deck until the first red card appears. For the following you may leave your answers in unsimplified form.

   X = # cards until first red card's

   $X = 1, 2, 3, \ldots, 27$

   (a) What is the probability that exactly 3 deals are required?

   (b) What is the probability that 3 or fewer deals are required?

(a) $P(\text{Exactly } 3 \text{ deals}) = P(BBR) =$

$= P(B) \cdot P(BB|B) \cdot P(BBR|BB)$ $= \boxed{\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{26}{50}}$ ✓

(b) $P(3 \text{ or fewer}) = 1 - P(4 \text{ or more}) = 1 - P(\text{first three black})$

$= \boxed{1 - \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50}}$ ✓ ; $P(3 \text{ or fewer}) = P(1) + P(2) + P(3)$

2. A fair coin is tossed n times. Given that at most 1 heads occurred, what is the probability that no heads occurred at all?

$P(\text{no Hs} \mid \text{at most } 1 H) = \frac{P(\text{no Hs} \cap \text{at most } 1 H)}{P(\text{at most } 1 H)} = \frac{P(\text{no Hs})}{P(\text{at most } 1 H)}$

$= \frac{P(\text{No Hs})}{P(\text{No Hs}) + P(1 H)} = \frac{\left(\frac{1}{2}\right)^n}{\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n} = \frac{\cancel{\left(\frac{1}{2}\right)^n}}{\cancel{\left(\frac{1}{2}\right)^n}(n+1)} \Rightarrow \boxed{\frac{1}{n+1}}$ ✓

n = number of tickets

3. Suppose that a randomly chosen airline passenger fails to show up for her flight with probability .04. What is the maximum number of tickets that the airline can sell for a flight with 200 seats, if it wants to be 95% sure that it will have seats for all the passengers that do show up? (Recall that $\Phi(-2, 2) = .95$.) Just set up the proper quadratic equation. Let X = "# of passengers that show". $X \sim Bin(n, .04)$

$P(X \le 200) = .95 \Leftrightarrow \sum_{i=0}^{200} \binom{n}{i} (.4)^i (.6)^{n-i} = .95$. Use Normal approx.

$P\left(\frac{X - \mu}{\sigma} \le \frac{200 - n \cdot p}{\sigma}\right) = .95 \Leftrightarrow P\left(Z \le \frac{200 - n \cdot p}{\sigma}\right) = .95$; $z_0 = \frac{200 - n \cdot p}{\sigma} = \frac{200 - np}{\frac{\sqrt{n}}{2}}$

$\Rightarrow z_0 = \frac{2(200 - np)}{\sqrt{n}} \Rightarrow z_0 \sqrt{n} = 400 - 2np$

4. In professional women's tennis, a match is won by the winner of 2 out of three sets. Suppose that two players, A and B, play a match, and player A has probability p of winning an individual set.

   (a) What is the probability (in terms of p and $q = 1 - p$) that player A wins the match?

   (b) What is the probability that the match goes three sets?

(a) $P(A \text{ wins}) = P(WW \text{ or } WLW \text{ or } LWW) = \boxed{p^2 + 2p^2 q}$

(b) $P(\text{match goes to three sets}) = P(ABA \text{ or } BAA \text{ or } BAB \text{ or } ABB)$

$= p^2 q + q p^2 + q^2 p + p q^2 = 2p^2 q + 2 p q^2 = \boxed{2pq(p+q)}$ ✓

$= \boxed{2pq}$ since $\boxed{p + q = 1}$