

## Probability distributions - summary

Discrete Distributions				
Distribution	Probability Mass Function	Mean	Variance	Moment-generating Function
Binomial	$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, \dots, n$	$np$	$np(1-p)$	$[pe^t + (1-p)]^n$
Geometric	$P(X = x) = (1-p)^{x-1} p$ $x = 1, 2, \dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Negative Binomial	$P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$ $x = r, r+1, \dots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$[\frac{pe^t}{1-(1-p)e^t}]^r$
Hypergeometric	$P(X = x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$ $x = 0, 1, \dots, n$ if $n \leq r$ , $x = 0, 1, \dots, r$ if $n > r$	$\frac{nr}{N}$	$n \frac{r}{N} \frac{N-r}{N} \frac{N-n}{N-1}$	Fairly complicated!
Poisson	$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$ $x = 0, 1, \dots$	$\lambda$	$\lambda$	$exp[\lambda(e^t - 1)]$
Continuous Distributions				
Distribution	Probability Density Function	Mean	Variance	Moment-generating Function
Uniform	$f(x) = \frac{1}{b-a}$ $a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$
Gamma	$f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\beta^\alpha \Gamma(\alpha)}$ , $\alpha, \beta > 0$ , $x \geq 0$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$
Exponential	$f(x) = \lambda e^{-\lambda x}$ , $\lambda > 0$ , $x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$(1 - \frac{1}{\lambda} t)^{-1}$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$ $-\infty < x < +\infty$	$\mu$	$\sigma^2$	$e^{\mu t + \frac{t^2 \sigma^2}{2}}$

Remarks:

- Binomial:  $X$  represents the number of successes among  $n$  trials.
- Geometric:  $X$  represents the number of trials needed until the first success.
- Negative Binomial:  $X$  represents the number of trials needed until  $r$  successes occur.
- Hypergeometric:  $X$  represents the number of items among the  $n$  selected that comes from the  $r$  group.
- Poisson:  $X$  represents the number of events that occur in time, area, etc.