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EXAM 2 M464 Prof. Lyons Spring 2014

Explain all answers. *If you use a formula or method we covered in class, you do not need to re-derive that formula here. Just state it clearly. If you find yourself doing long calculations, then you are doing the problem a hard way or incorrectly.*

1. (25 points) Let $\{X(t)\,;\ t \geq 0\}$ be a Poisson process of rate 4. Calculate the following (exactly, not numerically):

(a) $P[X(1) = 2]$.

(b) $P[X(1) = 2 \text{ and } X(3) = 6]$.

(c) $P[X(1) = 2 \mid X(3) = 6]$.

(d) $E[X(2)]$.

2. (15 points) Suppose that $5n$ points are uniformly and independently distributed in a circular disk of radius $\sqrt{n}$. Determine the limiting probability distribution of the number of points within distance 1 of the center of the disk as $n \to \infty$.

3. (15 points) For $i = 1, 2, 3$, let $\{X_i(t)\,;\ t \geq 0\}$ be independent Poisson processes, where the rate of the ith process is i^2. Let T be the first time that at least one event has occurred in each of the three processes. What is the distribution of T?

4. (25 points) Let $\{X(t)\,;\ t \geq 0\}$ be a Poisson process of rate 5. Let $W_1, W_2, \ldots$ be the arrival times.

(a) Calculate the probability that there is no arrival in the interval $[1/4, 3/4]$. (This is the event that for every $i \geq 1$, we have $W_i \notin [1/4, 3/4]$.)

(b) For each $n = 1, 2, 3, \ldots$, calculate the conditional probability that there is no arrival in the interval $[1/4, 3/4]$ given that $X(1) = n$.

(c) For each $n = 1, 2, 3, \ldots$, calculate the conditional probability that there is no arrival in the set $[0, 1/4] \cup [3/4, 1]$ given that $X(1) = n$. (This set is the union of two intervals; the event is that there is no arrival in either interval.)

(d) For each $n = 1, 2, 3, \ldots$, calculate $P\big[\min\{W_1, 1 - W_n\} > 1/4 \mid X(1) = n\big]$.

(e) For each $n = 1, 2, 3, \ldots$, calculate the conditional distribution of $\min\{W_1, 1 - W_n\}$ given that $X(1) = n$.

5. (20 points) Suppose that acorns [a type of nut] fall from an oak tree according to a Poisson process of rate 1 per hour. Also, suppose that squirrels [a ground animal] pass by the oak tree according to a Poisson process of rate 2 per hour, independently of the acorns. An arriving squirrel takes all acorns it finds, if any, and moves on. What is the probability that the second squirrel to arrive at the oak tree finds exactly two acorns?