S520 Homework 2

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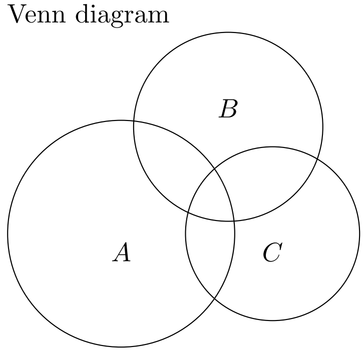
Chapter 3, Section 7, #1:

(a) Venn diagram



(b) By simple inspection of the Venn diagram, one can see that the probability is P(A and B) - P(A and B and C), i.e., subtract from P(A and B) those possibilities where C is also found.

$$P(A \cap B) - P(A \cap B \cap C) = 0.29 - 0.13 = 0.16$$

(c) We want to find only A.

$$P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) = 0.8 - 0.29 - 0.24 + 0.13 = 0.27 + 0.13 = 0.40$$

(d) $P(A^C \cap B^C \cap C^C) = P((A \cup B \cup C)^C) = 1 - P(A \cup B \cup C) =$
$1 - (P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) +$
$P(A \cap B \cap C)) =$
$1 - (0.8 + 0.36 + 0.28 - 0.29 - 0.24 - 0.16 + 0.13) =$
$1 - (1.44 - 0.69 + 0.13) = 1 - 0.88 = 0.12$

(e) $P(A^c \cap (B \cup C)) = P(B \cup C) - P(B \cap A) - P(C \cap A) + P(A \cap B \cap C) =$
$P(B) + P(C) - P(B \cap C) - P(B \cap A) - P(C \cap A) + P(A \cap B \cap C) =$
$0.36 + 0.28 - 0.16 - 0.29 - 0.24 + 0.13 = 0.08$

Chapter 3, Section 7, #4: The sample space for this experiment can be defined as:

$$S = \{(x_1, x_2, ..., x_5) : x_i \text{ is one of the 13 models, for } 1 \leq i \leq 5\}$$

This is a finite sample space: $|S| = P(13, 5) = 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 = 154,440$

(a) The event we are dealing with is:

$$A = \{(x_1, x_2, ..., x_5) : x_i \text{ is a finalist, for} 1 \leq i \leq 5\}$$

Because this is a finite sample space with equally likely probabilities, this problem reduces to a counting problem. We want to count the number of 5-tuples containing the finalist and divided by the total number of 5 tuples.

There are two ways to frame this problem: the number of ways of obtaining the finalist is the way of getting them in any order, i.e. $P(5,5)$, thus $P(A) = \frac{\#A}{\#S} = \frac{P(5,5)}{P(13,5)} = \frac{5\cdot4\cdot3\cdot2\cdot1}{13\cdot12\cdot11\cdot10\cdot9} = \frac{1}{13\cdot11\cdot3\cdot3} = \frac{1}{1287}$

Another way would be to count the 5-tuples but not considering order, i.e. $\frac{1}{\binom{13}{5}} = \frac{1}{\frac{13!}{8!5!}} = \frac{8!5!}{13!} = \frac{5!}{P(13,5)}$. In this case $\#A = 1$ because there is only one way of getting the 5 finalist without considering the order.

(b) The event we are looking at is:

$$B = \{(x_1, x_2, ..., x_5) : \exists x_i : x_i = \text{Saleisha, for} 1 \leq i \leq 5\}$$

The event B can be seen as the union of the events where Saleisha is draw first, second, third, fourth or fifth. This events are disjoints, so: $P(B) = \frac{5\cdot P(12,4)}{P(13,5)} = \frac{5}{13}$.

Chapter 3, Section 7, #8: $P(+|D) = 0.62$ and $P(-|D^c) = 0.82$

(a) False positive: $P(+|D^c) = 1 - (-|D^c) = 1 - 0.82 = 0.18$

(b) False negative: $P(-|D) = 1 - (+|D) = 1 - 0.62 = 0.38$

(c)

$$+ \ P(+|D) \longrightarrow P(+\cap D) = P(D)\cdot P(+|D) = 0.05\cdot 0.62 = 0.031$$

$$D \qquad - \ P(-|D)$$

$$D \qquad + \ P(+|D^c) \longrightarrow P(+\cap D^c) = P(D^c)\cdot P(+|D^c) = 0.95\cdot 0.18 = 0.171$$

$$- \ P(-|D^c)$$

(d) $P(+) = P(+\cap D) + P(+\cap D^c) = 0.031 + 0.171 = 0.202$

(e) $P(D|+) = \frac{P(+\cap D)}{P(+)} = \frac{P(+|D)P(D)}{P(+)} = \frac{0.62\cdot 0.05}{0.202} = 0.1534$

Chapter 3, Section 7, #11:

(a) Let $A =$ the event that the person is a male, and $B =$ the event that the person is struck by lightning. Then, $P(A|B) =$ is the probability of being male given that the person was struck by lightning.
By the data given on the problem, we can estimate $P(A|B) = 0.85$.
Therefore, $P(A^c|B) = 1 - P(A|B) = 1 - 0.85 = 0.15$

(b) We can estimate that $P(A)$ is approximately equal to 0.5, i.e., there are roughly the same number of males and females in the U.S. It follows that, $P(A|B) \not\approx P(A)$ and thus, these events are not independent. Intuitively, we can say that if we know that a person has been struck by lightning, then there is a higher chance this person is a male.

(c) Historically, man have undertaken outdoors activity (e.g., farming, fishing, etc) more so than women, who tended to stay indoor or at home more. Given that lightning usually struck a person that is not inside a building, more man have been at risk of being struck than women. However, as culture changes, and women take similarly responsibilities as man, maybe we can see a change in this tendency in the future.

Chapter 3, Section 7, #14:

(a) $P(X > 0.5) = 1 - P(X \leq 0.5) = 1 - F(0.5) = 1 - \frac{0.5}{3} = 1 - \frac{1}{6} = \frac{5}{6}$

(b) $P(2 < X \leq 3) = P(X \leq 3) - P(X \leq 2) = F(3) - F(2) = 1 - \frac{2}{3} = \frac{1}{3}$

(c) $P(0.5 < X \leq 2.5) = P(X \leq 2.5) - P(X \leq 0.5) = F(2.5) - F(0.5) = \frac{2.5}{3} - \frac{0.5}{3} = \frac{2}{3}$

(d) $P(X = 1) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ (The jump on the setp function on $X = 1$ from $\frac{1}{3}$ to $\frac{2}{3}$

Chapter 3, Section 7, #15:

(a) $P(X_2 \neq 7 \wedge X_2 \neq 8) =$
$1 - P(X_2 = 7 \vee X_2 = 7) = 1 - (P(X_2 = 7) + P(x_2 = 8)) =$
$1 - (\frac{6}{36} + \frac{5}{36}) = 1 - \frac{11}{36} = \frac{25}{36}$

(b) The key observation here is that X_2 and X_3 are independent and have the same numerical value, thus the probability is: $(\frac{25}{36})^2$

(c) Same reasoning as before: $(\frac{25}{36})^3$

(d) $P(\text{The shooter will never roll another 7 or 8}) = lim_{n\to\infty}(\frac{25}{36})^n = 0$. Eventually the shooter will roll a 7 or 8.