INFO I201
Sample Midterm II

Not to be collected.

Q1. Let $A = \mathbb{Z}$ (the set of integers), $B = \{0, 1\}$.

(a) Find $A \cap B, B - A, A \cup B$.

(b) Find $\mathcal{P}(B)$ (the power set of B).

(c) List two elements in $\mathcal{P}(A)$?

(d) Find $\mathcal{P}(A) \cap B$.

(e) List 4 elements in the set $A \times B$?

Q2. Let $\mathcal{L}$ be a first order language with one unary predicate symbol S, one binary predicate symbol T, and one binary function symbol P. Consider the formulas

- $\phi_1 = \forall x \forall z[(x \neq z) \longrightarrow \exists y(\neg(T(x, y) \wedge T(z, y)))]$
- $\phi_2 = \forall x \forall y \exists z[z \neq P(x, P(x, y))]$
- $\phi_3 = \forall x[\forall z(\exists y(T(x, y) \longrightarrow T(z, y)) \wedge S(x)) \vee T(y, z)]$

Determine the free and bound occurrences of all variables in $\phi_1.\phi_2$, and ϕ_3.

Q3. (i) Let A, B and C be sets. Show that $(A - B) - C \subseteq A - C$.

(ii) Let A, B and C be sets. Prove or disprove: $A \cap C = B - C$ implies that $A = B$.

(iii) Let A, B and C be sets. Prove or disprove: $A - C \neq B \cap C$ implies that $A \neq B$.

Q4. Consider a first order language $\mathcal{L}$ that consists of two unary predicate symbols P and Q and one binary predicate symbol S. Also consider the following formulas in this language:

- $\phi_1 \equiv \forall y \exists x S(y, x) \wedge \exists x \forall y S(x, y)$
- $\phi_2 \equiv (\exists x P(x) \wedge \exists x Q(x)) \longrightarrow \exists x(P(x) \wedge Q(x))$

Recall that a model M is a pair $M = (U, I)$, let $U = \{a, b, c, d\}$. Find one model $M_1 = (U, I_1)$ making ϕ_1 true and another model $M_2 = (U, I_2)$ making ϕ_2 false. Be precise in your reasoning and explain your answer.

Q5. Consider a first order language $\mathcal{L}$ that consists of one unary predicate symbol Q. Let ϕ be the formula:

$$\forall x \forall y \forall z[(Q(x) \wedge Q(y) \wedge Q(z)) \longrightarrow ((z = y) \vee (z = x) \vee (x = y))]$$

(i) Let $U = \{a, b, c, d\}$ and $I(Q) = \{a, b\}$. Is ϕ valid in this model?

(ii) Let $U' = \{a, b, c\}$ and $I'(Q) = \{a\}$. Is ϕ valid in this model?