B501 Assignment 2 Part II

Due Date: Monday, February 6, 2012
Due Time: 11:00pm

Please note that the lemma and theorem in the following questions are from the book *Introduction to the Theory of Computation, Second Edition.*

1. (15 points) (Taken from last assignment) Construct DFA's equivalent to the NFA's.

   (a) $(\{p, q, r, s\}, \{0, 1\}, \delta_1, p, \{s\})$

   (b) $(\{p, q, r, s\}, \{0, 1\}, \delta_2, p, \{q, s\})$

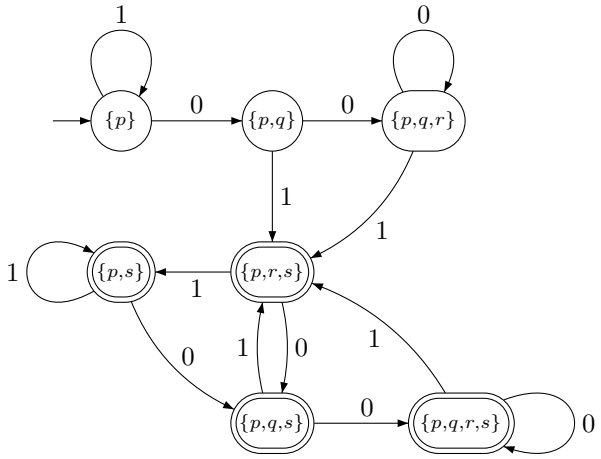
where δ_1 is given as

	0	1
p	p, q	p
q	r	r, s
r	$\emptyset$	s
s	s	s

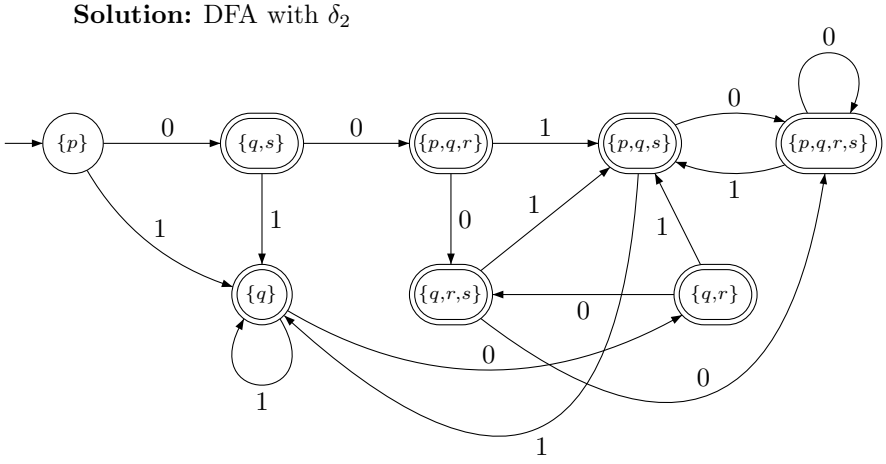
and δ_2 is given as

	0	1
p	q, s	q
q	q, r	q
r	s	p, s
s	p	$\emptyset$

Solution: DFA with δ_1



Solution: DFA with δ_2

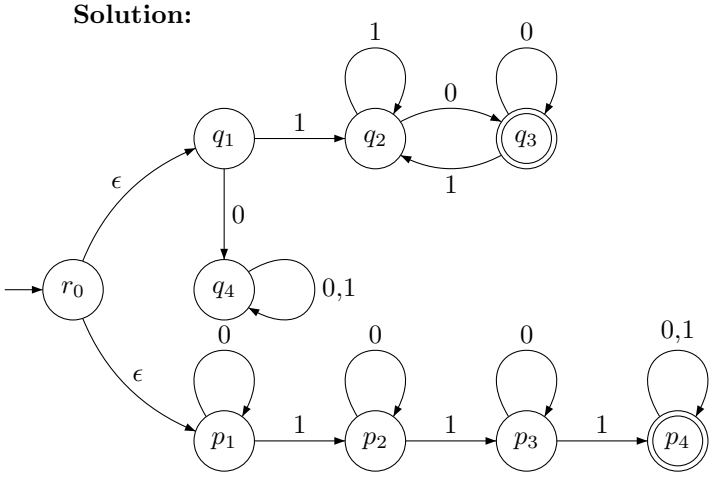


2. (10 points) Use the construction given in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages described as follows:

   (a) $\{w|w$ begins with a 1 and ends with a 0$\}$

   (b) $\{w|w$ contains at least three 1s$\}$

Solution:



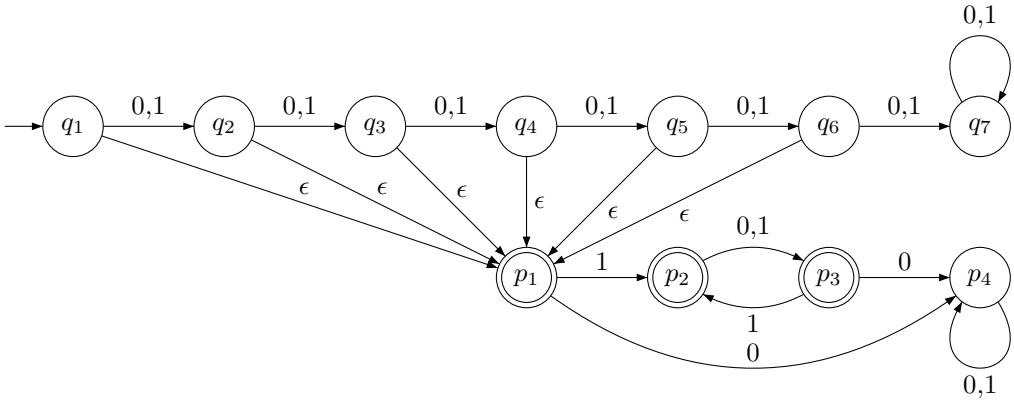
Note: I relabeled the states of the second machine so that they do not overlap states on the first machine.

3. (10 points) Use the construction given in the proof of Theorem 1.47 to give the state diagrams of NFAs recognizing the concatenation of the language described in follows:

   (a) $\{w|$ the length of w is at most 5$\}$

(b) $\{w|$ every odd position of w is 1$\}$

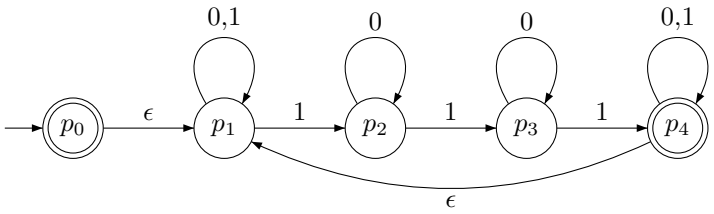
The DFAs for the languages are given to you as follows:



Note: I relabeled the states of the second machine so that they do not overlap states on the first machine.

4. (10 points) Use the construction given in the proof of Theorem 1.49 to give the state diagrams of NFAs recognizing the star of the language described in $\{w|w$ contains at least three 1s$\}$. The DFA for the language is already given in 2(b)

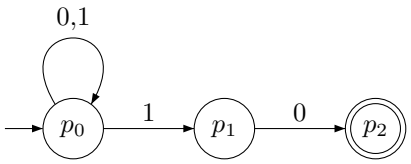
   Solution:



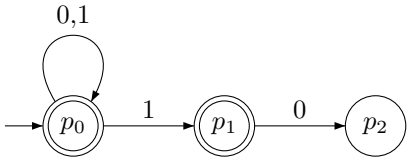
5. (10 points) Show by giving an example that, if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M doesn't necessarily yield a new NFS that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

   Solution:

   Let M be the NFA with alphabet $\Sigma = \{0, 1\}$ and with the following diagram:



Thus, $L(M) = \{w|w \text{ end in } 10\}$. Now, let M' be the NFA over the same alphabet and with the following diagram:

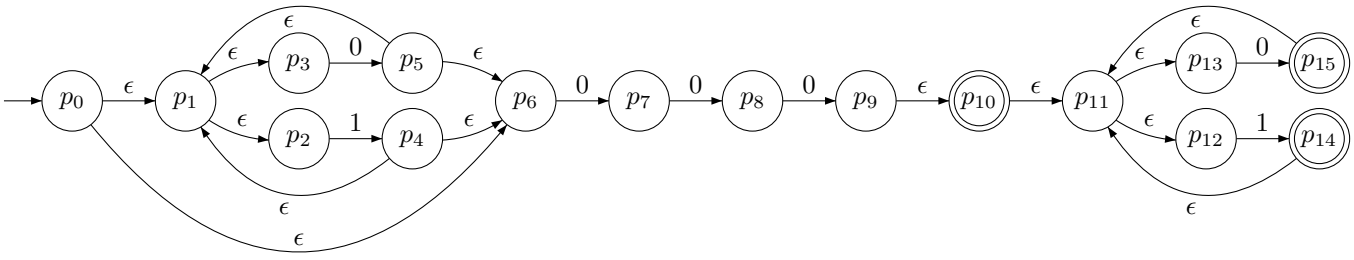


From the diagram we can see that M' is the automaton obtained by swapping the accept and nonaccept states in M.
However, $L(M') \neq L(M^c) = \{w|w \text{ does not end in } 10\}$. An easy way to see that this is the case, one can reason that the string 1110 is accepted by both M and M'.
Is the class of languages recognized by NFAs closed under complement?
Yes, it is. Let A be a regular language that is recognized by a NFA N, i.e., $L(N) = A$. For that machine we can construct a DFA N', such that $L(N') = A$, using theorem 1.39 (a.k.a power set construction). Swapping the accept/non-accept states of N', we obtain a DFA N'' that accepts the complement language of A, i.e. $L(N'') = A^c$. Finally, we can consider machine N'' to be a NFA and thus, we conclude that there exists a NFA N'' that accepts language A^c. The closure property holds.

6. (15 points) Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

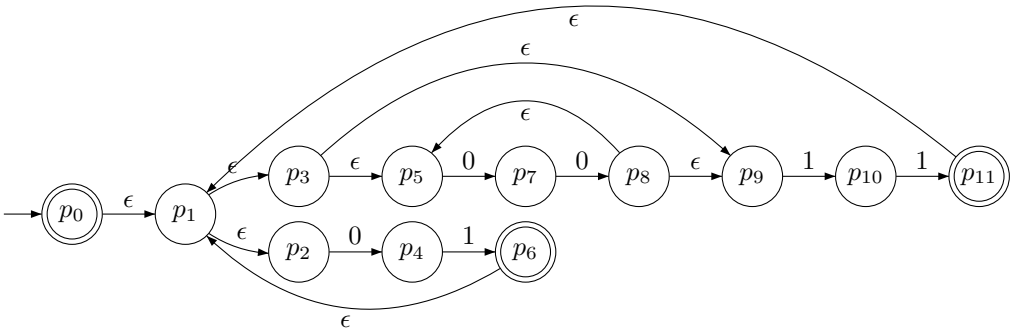
   (a) $(0 \cup 1)^*000(0 \cup 1)^*$

   Solution:



   (b) $(((00)^*(11)) \cup 01)^*$

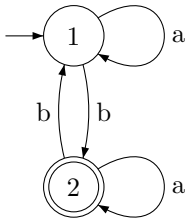
   Solution:



(c) $\emptyset^*$

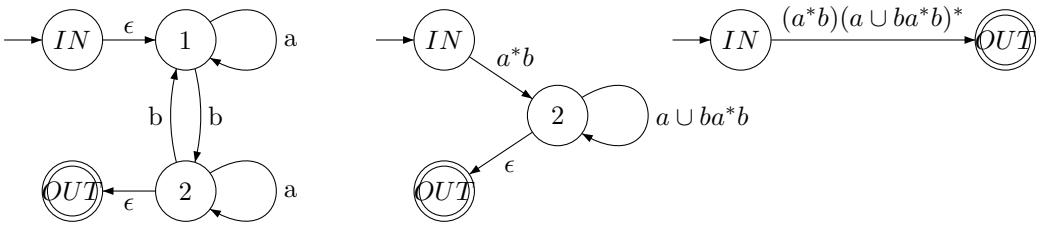
7. (10 points) Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.

(a)

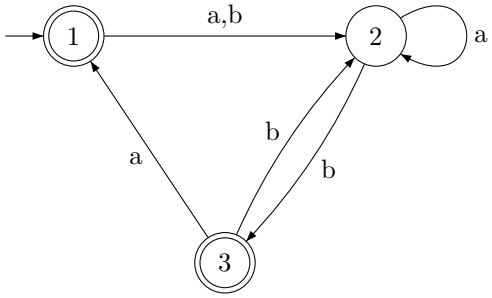


Solution:

First I normalize the NFA. Then I take state 1 out, then state 2.



(b)



Solution:

First I normalize the NFA. Then I take state 1 out, then state 2 and finally state 3.