B501 Assignment 2 Part I
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Due Date: Friday, January 27, 2012
Due Time: 11:00pm

1. (15 points) Let M be the finite automaton $(Q, \Sigma, \delta, q_0, F)$. Define the function
$\delta^* : Q \times \Sigma^* \to Q$ as follows:

   - $\delta^*(q, \varepsilon) = q$
   - $\delta^*(q, wa) = \delta(\delta^*(q, w), a)$, where $w \in \Sigma^*$ and $a \in \Sigma$

   (Recall that $L(M) = \{w \in \Sigma^* | \delta^*(q_0, w) \in F\}$, so δ^* is the recursive transition function of M.)
   Prove that for each x and y in Σ^*,

   $$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$$

   Hint: Use structural induction.

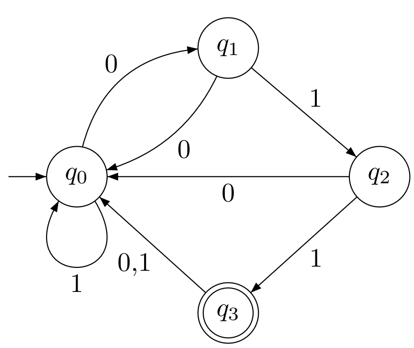
   Proof:

   1. Base Case: let $w = ee$ (the empty string followed by the empty string). Then, $\delta^*(q, w) = \delta^*(q, ee) = \delta(\delta^*(q, e), e) = \delta(q, e) = q$
   On the other hand, $\delta^*(\delta^*(q, e), e) = \delta^*(q, e) = q$. It holds.
   2. Inductive step: let $z = xya$, where $x, y \in \Sigma^*$ and $a \in \Sigma$. Then,

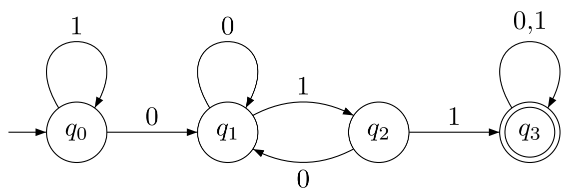
   $$\begin{aligned}
   \delta^*(q, z) &= \delta^*(q, xya) && \text{definition of z} \\
   &= \delta(\delta^*(q, xy), a) && \text{definition of } \delta^* \\
   &= \delta(\delta^*(\delta^*(q, x), y), a) && \text{hypothesis} \\
   &= \delta^*(\delta^*(q, x), ya) && \text{definition of } \delta^*\text{. Q.E.D.}
   \end{aligned}$$

2. (15 points) Give deterministic finite automata accepting the following languages over the alphabet (0,1).

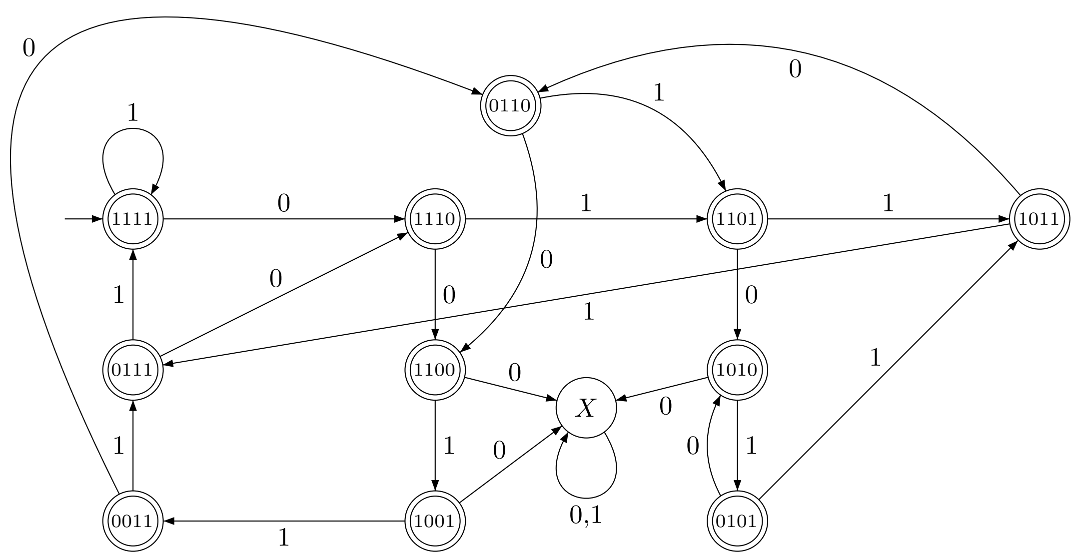
   (a) The set of all strings ending in 011.
   Solution:



(b) The set of all strings with "011" as a substring.
Solution:

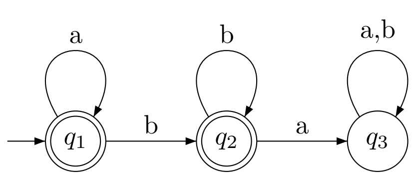


(c) The set of all strings such that every block of 4 consecutive symbols contains at least two 1's.
Solution: each state represent the memory of the previous last four characters read by the machine.



All state are finals except for $X = \{1000, 0010, 0100\}$.

3. (5 points) Describe in English the sets accepted by the following DFA.

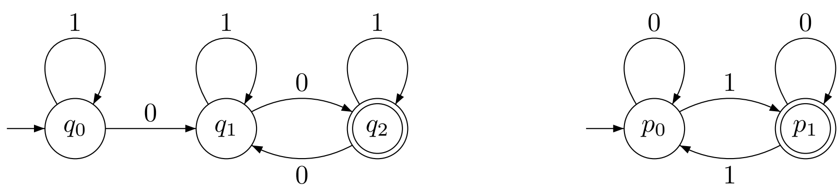


Solution: The DFA accepts the empty string, any number of a's and any number of a's followed by at least one b.

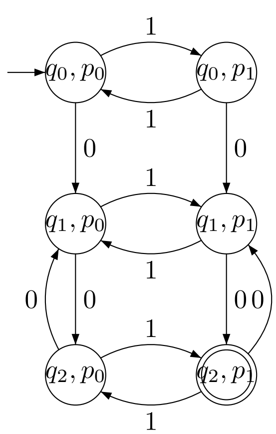
4. (15 points) Let $\Sigma = \{0, 1\}$, and let L be the set of strings that contain an even number of 0's and an odd number of 1's. Use the product construction to design a DFA that accepts L. (Draw appropriate diagrams)

   Solution:
   Let M_1 be the DFN such that $L(M_1) = \{w|\text{w contain an even number of 0's}\}$, and let M_2 be the DFN such that $L(M_2) = \{w|\text{w contain an odd number of 1's}\}$, Then:

   $M1 :=$ $M_2 :=$



   The product construction N of M_1 and M_2 is:

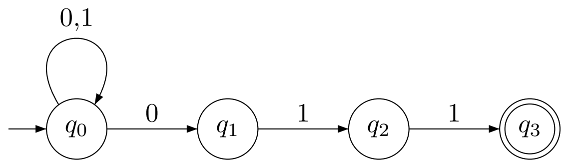


   This machines now accepts $L(N) = L(M_1) \cap L(M_2)$

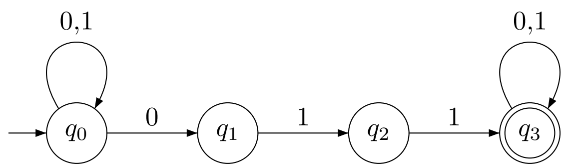
5. (15 points) Give nondeterministic finite automata accepting the languages given in problem 2. Make sure that when possible, you should design simpler automata than what you have for problem 2.

   (a) NFA accepting the set of all strings ending in 011.

Solution:



(b) NFA accepting the set of all strings with "011" as a substring.
Solution:



(c) NFA accepting The set of all strings such that every block of 4 consecutive symbols contains at least two 1's.
Solution:
Any DFA can be thought of as a NFA. All we have to do is expand Σ to include ϵ, and build a new transition function δ_n that maps the states appropriately according to these rules

$$\delta_n : Q \times (\Sigma \cup \{\epsilon\}) \mapsto 2^Q$$

$$\delta_n(q, a) = \{\delta(q, a)\}$$

$$\delta_n(q, \epsilon) = \{q\}$$

Thus, the DFA given in solution 2 (c) is also the NFA for this problem.

6. (10 points) Give nondeterministic finite automaton accepting the following language: The set of strings in $(\mathbf{0}+\mathbf{1})^*$ such that some two 1's are separated by a string whose length is $3i$, for some $i \geq 0$.
Solution: