**Homework 4 for M312, Section 30353**
**due Wednesday, September 25, 2013**

**1.** (10 pts) For which $a \in \mathbb{R}$ the vector field $F(x,y) = (e^{x+y} + ay, e^{x+y} + x)$ is a gradient vector field on $\mathbb{R}^2$? For those $a$ find a scalar function $f$ with $F = \nabla f$.
**2.** (10 pts) Exercise 7.2.11 (p. 374).
**3.** (10 pts) Exercise 7.2.14 (p. 374).
**4.** (5 pts) For a continuous vector field $F$ on a path $c$ show that

$$\left| \int_c F \cdot ds \right| \leq \int_c ||F \circ c|| ds.$$

**5.** (extra credit, 20 pts) For $(x,y) \neq (0,0)$ define the vector field

$$F(x,y) = \frac{1}{e^y \sqrt{x^2+y^2}} (\cos x, \sin x).$$

For $R > 0$ let $c(t) = R(\cos t, \sin t)$, $0 \leq t \leq \pi/2$ be a parametrization of a quartier if the circle centered at the origin with radius $R$. Use Problem 4 to prove that

$$\lim_{R \to \infty} \int_c F \cdot ds = 0.$$

**6.** (5 pts) Exercise 7.2.7 (p. 373).
**7.** (10 pts) Exercise 7.2.17 (p. 374).
**8.** (10 pts) Exercise 7.2.18 (p. 374).
**9.** (10 pts) Exercise 7.3.3 (p. 381).
**10.** (10 pts) Exercise 7.3.4 (p.381).
**11.** (10 pts) Exercise 7.3.9 (p. 382).
**12.** (10 pts) Exercise 7.3.14 (p. 382).