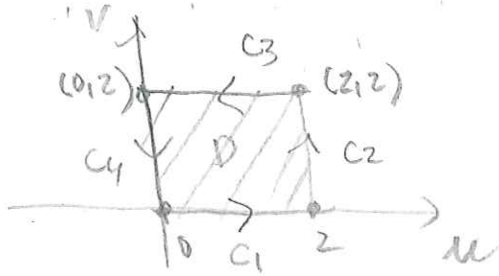
M312 - Fall 2013 - HW 11 - Enrique Areyan (100) (1)

(1) Exercise 8.2.2. Let S be the portion of the surface $z=x^2+y^2$ lying between the points $(0,0,0)$, $(2,0,4)$, $(0,2,4)$, and $(2,2,8)$. Find parametrizations for both the surface S and its boundary ∂S. Be sure that their respective orientations are compatible with Stoke's theorem.

Solution: the parametrization for the surface (since z is a graph) $\Phi(u,v)=(u,v,u^2+v^2)$. Now we need to find the domain D of point (u,v) allowed for our surface: Projecting the lines we get



$$D=\{(u,v):\ 0\le u\le 2,\ 0\le v\le 2\}.$$

Now, for the parametrization of the boundary, we parametrize each piece:

$$c_1(t)=(t,0),\quad 0\le t\le 2$$
$$c_2(t)=(2,t),\quad 0\le t\le 2$$
$$c_3(t)=(2-t,2),\quad 0\le t\le 2$$
$$c_4(t)=(0,2-t),\quad 0\le t\le 2$$

$$\begin{cases} \Phi(c_1(t))=\Phi(t,0)=(t,0,t^2);\ 0\le t\le 2 \\ \Phi(c_2(t))=\Phi(2,t)=(2,t,t^2+4);\ 0\le t\le 2 \\ \Phi(c_3(t))=\Phi(2-t,2)=(2-t,2,2^2+(2-t)^2)=(2-t,2,t^2-4t+8),\ 0\le t\le 2 \\ \Phi(c_4(t))=\Phi(0,2-t)=(0,2-t,(2-t)^2)=(0,2-t,t^2-4t+4),\ 0\le t\le 2 \end{cases}$$

Bandary.

(2) Exercise 8.2.3. Verify Stoke's theorem for the given surface S and boundary ∂S, and vector field F.

$S = \{(x,y,z): x^2+y^2+z^2=1,\ z \geq 0\}$ (oriented as a graph)

$\partial S = \{(x,y): x^2+y^2=1\}$

$\vec{F} = \langle x, y, z \rangle$.

We want to verify: $\iint_S (\nabla \times F) \cdot d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{s}$.

Let us first compute the left-hand side:

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(0) = \langle 0,0,0 \rangle.$$ Hence,

$$\iint_S (\nabla \times F) \cdot d\vec{S} = \iint_S 0 \, d\vec{S} = \boxed{0}$$

Now, let us compute the right-hand side:

A parametrization of the boundary is $c(t) = \langle \cos t, \sin t, 0 \rangle$ $0 \leq t \leq 2\pi$

$$\int_{\partial S} \vec{F} \cdot d\vec{s} = \int_C \vec{F}(c(t)) \cdot c'(t)\, dt = \int_0^{2\pi} \vec{F}(\cos t, \sin t, 0) \cdot \langle -\sin t, \cos t, 0 \rangle\, dt$$

$$= \int_0^{2\pi} \langle \cos t, \sin t, 0 \rangle \cdot \langle -\sin t, \cos t, 0 \rangle\, dt = \int_0^{2\pi} -\cos t \sin t + \cos t \sin t\, dt = \int_0^{2\pi} 0\, dt = \boxed{0}$$

(3) Exercise 8.2.4. SAME AS before but with: $F = \langle y, z, x \rangle$.

Let us first compute the left-hand side:

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix} = \hat{i}(-1) - \hat{j}(1) + \hat{k}(-1) = \langle -1, -1, -1 \rangle$$

A parametrization of S as a graph is:

$\Phi(u,v) = (u, v, \sqrt{1-u^2-v^2})$; $u^2+v^2 \leq 1$ ← This is not the surface considered.

$$\Phi_u = \left(1, 0, \frac{-u}{\sqrt{1-u^2-v^2}}\right) \quad \Phi_u \times \Phi_v = \hat{i}\left(\frac{+u}{\sqrt{1-u^2-v^2}}\right) + \hat{j}\left(\frac{v}{\sqrt{1-u^2-v^2}}\right) + \hat{k}(1) = \left\langle \frac{u}{\sqrt{1-u^2-v^2}}, \frac{v}{\sqrt{1-u^2-v^2}}, 1 \right\rangle$$

$$\Phi_v = \left(0, 1, \frac{-v}{\sqrt{1-u^2-v^2}}\right) \quad \|\Phi_u \times \Phi_v\| = \sqrt{\frac{u^2+v^2+1-u^2-v^2}{1-u^2-v^2}} = \frac{1}{\sqrt{1-u^2-v^2}}$$

$$\iint_S \nabla \times \vec{F} \cdot d\vec{S} = \iint_S \langle -1,-1,-1 \rangle \cdot d\vec{S},$$ Now $(\nabla \times \vec{F})(\Phi(u,v)) = \langle -1,-1,-1 \rangle$; Hence →

$$\iint_S \langle -1,-1,-1\rangle \cdot d\vec{S} = \iint_S \langle -1,-1,-1\rangle \cdot (\Phi_u \times \Phi_v)\, dS = \iint_S \langle -1,-1,-1\rangle \cdot \left\langle \frac{u}{\sqrt{1-u^2-v^2}}, \frac{v}{\sqrt{1-u^2-v^2}}, 1\right\rangle dS$$

$$= \iint_S -\frac{u}{\sqrt{1-u^2-v^2}} - \frac{v}{\sqrt{1-u^2-v^2}} - 1\, dS = \iint_{u^2+v^2\leq 1} \left(-\frac{u}{\sqrt{1-u^2-v^2}} - \frac{v}{\sqrt{1-u^2-v^2}} - 1\right) \frac{1}{\sqrt{1-u^2-v^2}}\, du\,dv$$

$$= \iint_{u^2+v^2\leq 1} \frac{-u-v-\sqrt{1-u^2-v^2}}{1-u^2-v^2}\, du\,dv = \iint_{u^2+v^2\leq 1} \frac{-u-v-1}{1}\, du\,dv = \iint_{u^2+v^2\leq 1} -u-v-1\, du\,dv.$$

changing to Polar: $\displaystyle\int_0^{2\pi}\int_0^1 (-r\cos\theta - r\sin\theta - 1)\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 -r^2\cos\theta - r^2\sin\theta - r\, dr\,d\theta$

$$= -\int_0^{2\pi} \left[\frac{r^3}{3}\cos\theta + \frac{r^3}{3}\sin\theta + \frac{r^2}{2}\right]_{r=0}^{r=1} = -\int_0^{2\pi} \frac{\cos\theta}{3} + \frac{\sin\theta}{3} + \frac{1}{2}\, d\theta = -\left[\frac{\sin\theta}{3} - \frac{\cos\theta}{3} + \frac{\theta}{2}\right]_{\theta=0}^{\theta=2\pi}$$

$$= -\left[\left(\frac{\sin(2\pi)}{3} - \frac{\cos(2\pi)}{3} + \frac{2\pi}{2}\right) - \left(\frac{\sin(0)}{3} - \frac{\cos(0)}{3} + \frac{0}{2}\right)\right] = -\left[0 - 1 + \pi - 0 + 1 - 0\right] = \boxed{-\pi}$$

Let us compute the right-hand side: $c(t) = (\cos t, \sin t, 0)$; $0 \leq t \leq 2\pi$

$$\int_{\partial S} \vec{F} \cdot d\vec{s} = \int_C \vec{F}(c(t)) \cdot c'(t)\, dt = \int_C \langle \sin t, 0, \cos t\rangle \cdot \langle -\sin t, \cos t, 0\rangle\, dt$$

$$= \int_C -\sin^2 t\, dt = \int_0^{2\pi} -\sin^2 t\, dt = -\frac{1}{2}\left[(t - \sin(t)\cos(t))\right]_0^{2\pi} = -\frac{1}{2}\left[2\pi - \sin(2\pi)\cos(2\pi) - 0 + \sin(0)\cos(0)\right]$$

$$= \boxed{-\pi}$$

(4) Exercise 8.2.6. Same as before but $\vec{F} = \langle z^2, x, y^2\rangle$

Solution: As usual, first let us compute the left-hand side:

$$\nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & x & y^2 \end{vmatrix} = \hat{i}(2y) - \hat{j}(-2z) + \hat{k}(1) = \langle 2y, 2z, 1\rangle.$$

Using same parametrization as before:

$$\iint_S (\nabla\times F)\cdot d\vec{S} = \iint_S \langle 2y, 2z, 1\rangle \cdot d\vec{S} = \iint_S \langle 2y, 2z, 1\rangle \cdot (\Phi_u \times \Phi_v)\, dS,$$ where

$\langle 2y, 2z, 1\rangle \cdot \langle u, v, \sqrt{1-u^2-v^2}\rangle = 2yu + 2zv + \sqrt{1-u^2-v^2} = G$, but then

$G(\Phi(u,v)) = 2uv + 2v\sqrt{1-u^2-v^2} + \sqrt{1-u^2-v^2} = 2uv + (2v+1)\left(\sqrt{1-u^2-v^2}\right) \longrightarrow$

$$\iint_S 2uv + (2v+1)\left(\sqrt{1-u^2-v^2}\right) dS = \iint_S 2uv + 2v + 1 \, dS, \text{ since } 1-u^2v^2=1.$$

$$= \iint_{u^2+v^2\le 1} 2uv+2v+1 \, \|\Phi_u \times \Phi_v\| \, du\,dv = \iint_{u^2+v^2\le 1} 2uv+2v+1 \, du\,dv. \text{ Changing to polar}$$

$$\int_0^{2\pi}\int_0^1 (2r^2\sin\theta\cos\theta + 2r\sin\theta + 1)\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^1 2r^3\sin\theta\cos\theta + 2r^2\sin\theta + r \, dr\,d\theta$$

$$= \int_0^{2\pi}\left[\frac{r^4}{2}\sin\theta\cos\theta + \frac{2}{3}r^3\sin\theta + \frac{r^2}{2}\right]_{r=0}^{r=1} d\theta = \int_0^{2\pi} \frac{\sin\theta\cos\theta}{2} + \frac{2\sin\theta}{3} + \frac{1}{2}\, d\theta$$

$$= \left[-\frac{1}{4}\cos^2(\theta) - \frac{2}{3}\cos\theta + \frac{1}{2}\theta\right]_{\theta=0}^{\theta=2\pi} = \left[\left(-\frac{1}{4}\cos^2(2\pi) - \frac{2}{3}\cos(2\pi) + \frac{2\pi}{2}\right) - \left(-\frac{1}{4}\cos^2(0) - \frac{2}{3}\cos(0) + \frac{0}{2}\right)\right]$$

$$= \left[\left(\cancel{-\frac{1}{4}} \cancel{-\frac{2}{3}} + \pi\right) - \left(\cancel{-\frac{1}{4}} \cancel{-\frac{2}{3}} + 0\right)\right] = \boxed{\pi}$$

Let us compute the right-hand side : $c(t) = (\cos t, \sin t, 0)$ $0 \le t \le 2\pi$

$$\int_{\partial S} \vec{F} \cdot d\vec{s} = \int_C \vec{F}(c(t)) \cdot c'(t)\,dt = \int_0^{2\pi} \langle 0, \cos t, \sin^2 t \rangle \cdot \langle -\sin t, \cos t, 0 \rangle \, dt$$

$$= \int_0^{2\pi} \cos^2 t \, dt = \left[\frac{1}{2}(t + \sin(t)\cos(t))\right]_{t=0}^{t=2\pi} = \left[\frac{1}{2}(2\pi + \overbrace{\sin(2\pi)}^{0}\cos(2\pi)) - \frac{1}{2}(0 + \overbrace{\sin(0)\cos(0)}^{0})\right]$$

$$= \boxed{\pi}$$

Both left and right-hand side agree.

(5) Exercise 8.2.7

Let C be the closed, piecewise smooth curve formed by traveling in straight lines between the points $(0,0,0), (2,0,4), (3,2,6), (1,2,2)$, and back to the origin, in that order. (thus, the surface S lying interior to C is contained in the plane $z=2x$). Use Stoke's theorem to evaluate

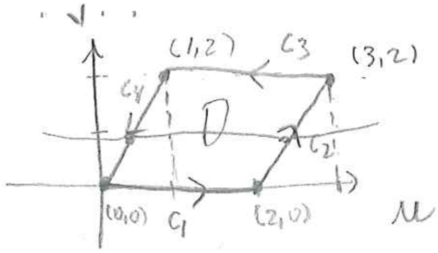
$$\int_C (z\cos x)dx + (x^2yz)\,dy + (yz)\,dz.$$

Solution : Stokes theorem states $\iint_S \nabla \times \vec{F} \, d\vec{S} = \int_{\partial S} \vec{F} \cdot d\vec{s}$. So let :

$$\vec{F} = (z\cos x, x^2yz, yz). \Rightarrow \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z\cos x & x^2yz & yz \end{vmatrix} = \hat{i}(z - x^2y) + \hat{j}(\cos x) + \hat{k}(2xyz)$$

$$= \langle z - x^2y, \cos x, 2xyz \rangle.$$

The parametrization of the surface is: $\Phi(u,v) = (u, v, 2u)$; the domain of this parametrization corresponds to the projection of the points in the xy-plane (same as uv-plane). $(0,0), (2,0), (3,2), (1,2)$



$C_1 \Rightarrow v=0$; $C_3 \Rightarrow v=2$

$C_2 \Rightarrow$ line containing the points $(2,0), (3,2)$.

$v = mu + b$: $\quad 2 = 3m + b \Rightarrow 2 = 6 + b \Rightarrow b = -4$

$0 = 2m + b$

$2 = m$

$C_2: v = 2u - 4$

$C_4 \Rightarrow$ line containing the points $(0,0), (1,2)$

$C_4: v = 2u$

So the domain D can be expressed as:

$D = \{ (u,v): \frac{v}{2} \le u \le \frac{v+4}{2},\ 0 \le v \le 2 \}$. Now we can compute

$$\iint_S \nabla \times \vec{F}\, d\vec{S} = \iint_D \langle z - x^2y, \cos x, 2xyz \rangle \cdot (\Phi_u \times \Phi_v)\, dA, \quad \text{where}$$

$\Phi_u = (1,0,2)$
$\Phi_v = (0,1,0)$
$\leadsto \Phi_u \times \Phi_v = \vec{i}(-2) - \vec{j}(0) + \vec{k}(1) = \langle -2, 0, 1 \rangle$

$$= \iint_D \langle z - x^2y, \cos x, 2xyz \rangle \cdot \langle -2, 0, 1 \rangle\, dA = \iint_D -2z + 2x^2y + 2xyz\, dA$$

$$= \iint_D -2(2u) + 2u^2v + 2uv(2u)\, dS = \iint_D -4u + 2u^2v + 4u^2v = \iint_D 6u^2v - 4u\, dA$$

$$= \int_0^2 \int_{\frac{v}{2}}^{\frac{v+4}{2}} 6u^2v - 4u\, du\, dv = \int_0^2 \left[2u^3v - 2u^2 \right]_{u=\frac{v}{2}}^{u=\frac{v+4}{2}} dv =$$

$$= \int_0^2 \left(2\left(\frac{v+4}{2}\right)^3 v - 2\left(\frac{v+4}{2}\right)^2 \right) - \left(2\left(\frac{v}{2}\right)^3 v - 2\left(\frac{v}{2}\right)^2 \right) dv$$

$$= \int_0^2 \left(\left[\frac{1}{4}(v^2+8v+16)(v+4)\right] - \frac{1}{2}\left[v^2+8v+16\right] \right) - \left(\frac{v^4}{4} - \frac{v^2}{2} \right) dv$$

$$= \int_0^2 \frac{1}{4}(v^3 + 4v^2 + 8v^2 + 32v + 16v + 64) - \frac{1}{2}(v^2+8v+16) - \frac{v^4}{4} + \frac{v^2}{2}\, dv$$

$$= \int_0^2 \frac{1}{4}(v^3 + 12v^2 + 48v + 64) - \frac{1}{2}(v^2+8v+16) - \frac{v^4}{4} + \frac{v^2}{2} = \int_0^2 \frac{v^3}{4} + 3v^2 + 44v + 56 - \frac{v^4}{4}\, dv$$

$$= \left[\frac{v^4}{16} + v^3 + 22v^2 + 56v - \frac{v^5}{20} \right]_0^2 = \frac{16}{16} + 8 + 22(4) + 56(2) - \frac{32}{20} = \boxed{52}$$

(6) Exercise 8.2.26. If C is a closed curve that is the boundary of a surface S, and f and g are C^2 functions, show that

(a) $\int_C f \nabla g \cdot d\vec{s} = \iint_S (\nabla f \times \nabla g) \cdot d\vec{S}$

Pf: By Stoke's theorem, it suffices to show that
$\nabla \times (f \nabla g) \stackrel{?}{=} (\nabla f \times \nabla g)$. So let us prove this equality:

$$\nabla \times (f\nabla g) = \nabla \times \left(\left\langle f\frac{\partial g}{\partial x}, f\frac{\partial g}{\partial y}, f\frac{\partial g}{\partial z}\right\rangle\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f\frac{\partial g}{\partial x} & f\frac{\partial g}{\partial y} & f\frac{\partial g}{\partial z} \end{vmatrix} = \hat{i}\left(\frac{\partial}{\partial y}\left(f\frac{\partial g}{\partial z}\right) - \frac{\partial}{\partial z}\left(f\frac{\partial g}{\partial y}\right)\right) - \hat{j}\left(\frac{\partial}{\partial x}\left(f\frac{\partial g}{\partial z}\right) - \frac{\partial}{\partial z}\left(f\frac{\partial g}{\partial x}\right)\right) + \hat{k}\left(\frac{\partial}{\partial x}\left(f\frac{\partial g}{\partial y}\right) - \frac{\partial}{\partial y}\left(f\frac{\partial g}{\partial x}\right)\right)$$

$$= \left\langle \frac{\partial f}{\partial y}\frac{\partial g}{\partial z} + \cancel{f\frac{\partial^2 g}{\partial z \partial y}} - \frac{\partial f}{\partial z}\frac{\partial g}{\partial y} - \cancel{f\frac{\partial^2 g}{\partial y \partial z}}, \frac{\partial f}{\partial z}\frac{\partial g}{\partial x} + \cancel{f\frac{\partial^2 g}{\partial x \partial z}} - \frac{\partial f}{\partial x}\frac{\partial g}{\partial z} - \cancel{f\frac{\partial^2 g}{\partial z \partial x}}, \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} + \cancel{f\frac{\partial^2 g}{\partial y \partial x}} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x} - \cancel{f\frac{\partial^2 g}{\partial x \partial y}} \right\rangle$$

; But $g \in C^2$, so mix partials are equal

$$= \left\langle \frac{\partial f}{\partial y}\cdot\frac{\partial g}{\partial z} - \frac{\partial f}{\partial z}\frac{\partial g}{\partial y}, \frac{\partial f}{\partial z}\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial g}{\partial z}, \frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x} \right\rangle$$

$$= \hat{i}\left(\frac{\partial f}{\partial y}\cdot\frac{\partial g}{\partial z} - \frac{\partial f}{\partial z}\frac{\partial g}{\partial y}\right) - \hat{j}\left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial z} - \frac{\partial f}{\partial z}\frac{\partial g}{\partial x}\right) + \hat{k}\left(\frac{\partial f}{\partial x}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial x}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \end{vmatrix} = \nabla f \times \nabla g \quad \square$$

(Result follows from Stoke's theorem).

(b) $\int_C (f\nabla g + g \nabla f) \cdot d\vec{s} = 0$.

Pf: $\int_C (f\nabla g + g\nabla f)\cdot d\vec{s} = \int_C f\nabla g \, d\vec{s} + \int_C g\nabla f \, d\vec{s}$ (linearity of integral)

$= \iint_S (\nabla f \times \nabla g)\cdot d\vec{S} + \iint_S (\nabla g \times \nabla f) d\vec{S}$ (By part (a))

$= \iint_S \nabla f \times \nabla g \cdot d\vec{S} - \iint_S \nabla f \times \nabla g \, d\vec{S}$ (Properties of cross product (orientation of surface integral)

$= 0$

(7) Verify the Mean-Value theorem for Harmonic Functions for

$u(x,y) = x^3 - 3xy^2$

this is harmonic because $u_{xx} + u_{yy} = (3x^2 - 3y^2)_x + (-6xy)_y$
$= 6x - 6x = \boxed{0}$

We want to check $u(0,0) \stackrel{?}{=} \frac{1}{2\pi} \int_{\partial D} u\,ds$, for our particular u, i.e.,

$u(0,0) = 0 \stackrel{?}{=} \frac{1}{2\pi} \int_{\partial D} u\,ds$, where $D = \{(x,y) : x^2 + y^2 \leq 1\}$.

So, let us compute: using parametrization $c(t) = (\cos t, \sin t)$ $0 \leq t \leq 2\pi$

$$\frac{1}{2\pi} \int_{\partial D} u\,ds = \frac{1}{2\pi} \int_C u(c(t))\, \|c'(t)\|\, dt = \frac{1}{2\pi} \int_0^{2\pi} \cos^3 t - 3\cos t \sin^2 t\, dt = (*)$$

By result used in class: $\int_0^{2\pi} \sin^p t + \cos^q t\, dt = 0$ if either p or q are odd

We can conclude that:

$$\frac{1}{2\pi}\left[\int_0^{2\pi} \cos^3 t - 3\cos t \sin^2 t\, dt\right] = \frac{1}{2\pi}\int_0^{2\pi} \cos^3 t\, dt - \frac{3}{2\pi}\int_0^{2\pi} \cos t \sin^2 t\, dt \quad \text{(apply result)}$$

$$= \frac{1}{2\pi} \cdot 0 - \frac{3}{2\pi} \cdot 0$$

$= \boxed{0}$ $\Rightarrow$ we have verified the mean-value theorem for $u(x,y) = x^3 - 3xy^2$

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(8) Use this theorem for $u(x,y) = e^x \cos y$ to compute

$$\int_0^{2\pi} e^{\cos t} \cos(\sin t)\, dt$$

Solution: By the mean-value theorem; using usual parame. $c(t) = (\cos t, \sin t)$ $0 \leq t \leq 2\pi$

$$u(0,0) = 1 = \frac{1}{2\pi} \int_{\partial D} e^x \cos(y)\, ds = \frac{1}{2\pi} \int_0^{2\pi} e^{\cos t} \cos(\sin t)\, dt$$

$$\Rightarrow \boxed{\int_0^{2\pi} e^{\cos t} \cos(\sin t)\, dt = 2\pi}$$

Note u is harmonic: $u_{xx} + u_{yy} = (e^x \cos y)_x + (e^x(-\sin y))_y = e^x \cos y - e^x \cos y = 0$

(9) Use this theorem to compute:

$$\int_0^{2\pi} \ln(5-4\cos t)\,dt.$$

solution: let $u(x,y) = \ln(5-4y)$ ← Is it harmonic?

the parametrization for the unit circle is $c(t) = (\cos t, \sin t)$ $0 \le t \le 2\pi$.

$$u(0,0) = \frac{1}{2\pi}\int_{\partial D} u\,ds = \frac{1}{2\pi}\int_0^{2\pi} \underbrace{\ln(5-4\cos t)}_{= u(c(t)) = u(\cos t, \sin t)}\,dt$$

X

$\Rightarrow$