MATH 343 Summer I 2013 12 May 21^{st} ,2013
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Quiz#2 Name: Enrique Areyan.

You have 20 minutes to finish the following 2 problems.

1. (*7 points*) Verify that the function y_1 and y_2 are solutions of the given O.D.E. Do they constitute a fundamental set of solutions?

$$y'' + 4y = 0, \qquad y_1(t) = \cos(2t), \quad y_2(t) = \sin(2t).$$

For $y_1(t)$ to be a solution we have to have:

$$y_1'' + 4y_1 = (\cos(2t))'' + 4(\cos(2t))$$
$$= (-\sin(2t)\cdot 2)' + 4\cos(2t))$$
$$= -4\cos(2t) + 4\cos(2t) = 0 \Rightarrow$$ y_1 is a sol.

For $y_2(t)$ to be a solution we have to have:

$$y_2'' + 4y_2 = (\sin(2t))'' + 4(\sin(2t))$$
$$= 2(\cos(2t))' + 4\sin(2t))$$
$$= -4\sin(2t) + 4\sin(2t) = 0 \Rightarrow$$ y_2 is a sol.

they will form a F.S.O.S iff $W(y_1, y_2)(t) \neq 0$ for some t.
Let us check:

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 \cdot y_2' - y_1' y_2 = \cos(2t)\cos(2t) - (-\sin(2t))\sin(2t)$$
$$= \sin^2(2t) + \cos^2(2t) = 1,$$

for any t.

Hence, these form a F.S.O.S for any t.

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(*) If $x=1 \Rightarrow \frac{-1\pm\sqrt{1}}{-4} = \frac{-1\pm1}{-4}$, this is zero when we add one, so the unique solution containing our point is:

$$\boxed{y = \frac{-x+\sqrt{24x^3-7x-16}}{-4}}$$

2. (*8 points*) Solve the initial value problem:

$$\underbrace{(9x^2+y-1)}_{M}dx - \underbrace{(4y-x)}_{N}dy = 0, \qquad y(1)=0$$

Let us check if this is an exact eq.

First, rewrite: $\underbrace{(9x^2+y-1)}_{M}dx + \underbrace{(x-4y)}_{N}dy = 0$

$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x} \Rightarrow$ this is an exact eq.

the solution is given by $\varphi(x,y) = C$, where $\frac{\partial \varphi}{\partial x} = M$ and $\frac{\partial \varphi}{\partial y} = N$ AND C is a constant

Hence,

$$\varphi(x,y) = \int \frac{\partial \varphi}{\partial y} \cdot \partial y = \int (x-4y)\partial y = xy - 2y^2 + h(x),$$

where $h(x)$ is a pure function of x. We find $h(x)$ as follow:

$$\frac{\partial \varphi}{\partial x} = M = 9x^2+y-1 = \frac{\partial}{\partial x}(xy-2y^2+h(x)) = y + h'(x)$$

$$\Rightarrow h'(x) = 9x^2-1 \Rightarrow h(x) = \int h'(x)\,dx = \int 9x^2-1\,dx = 3x^3 - x$$

the solution is: $\varphi(x,y) = \boxed{C = xy - 2y^2 + 3x^3 - x}$ in implicit form.

Solving for the initial condition $y(x=1)=0$:

$C = 1\cdot 0 - 2(0)^2 + 3(1)^3 - 1 = 2$

$\Rightarrow \boxed{C=2}$ the solution to the I.V.P is $\boxed{xy - 2y^2 + 3x^3 - x = 2}$

the solution in explicit form is: (using quadratic eq $-2y^2 + (x)y + (3x^3-x-2) = 0$

$$y = \frac{-x \pm \sqrt{x^2+8(3x^3-x-2)}}{-4} = \frac{-x \pm \sqrt{x^2+24x^3-8x-16}}{-4} = \frac{-x \pm \sqrt{24x^3-7x-16}}{-4} = (*)$$

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