Homework - MS403

Due Tuesday, December 10, 2013

Remember to write on only one side of the sheet.

1. Determine generators and relations for the group D_n, using two generators and three relations.

2. (a) Determine generators and relations for the quaternion group Q_8.

(b) Compute $|Aut(Q_8)|$.

3. Identify the following group: $G =< x, y, z | x^2 = y^2 = z^2 = e, xz = zx, xyx = yxy, yzy = zyz >$.

4. Determine the automorphism group of D_4.

5. Let G be a group and H a subgroup. We have seen that G acts on G/H, the set of left cosets of H in G, by left multiplication, and that we obtain therefore a homomorphism $\alpha : G \to A(G/H)$.

(a) Prove that if $N \subseteq H$ is a normal subgroup of G, then $N \subseteq ker(\alpha)$. (This is the sense in which, as I said in class, the kernel of α is the "largest" normal subgroup of G contained in H.)

(b) Now assume G is finite and let $n = [G : H]$, so we may think of α as a homomorphism from G to S_n. Prove that $\alpha(H) \subset S_{n-1}$. (This statement is not quite precise, but if you figure out what is going on, you will understand it.)

(c) Now assume G is finite and assume G has a subgroup H whose index in G is the smallest prime that divides the order of G. Prove H is normal in G. (This generalizes the result that if H is a subgroup of index 2, then H is normal.)

6. Find a Sylow-p subgroup in $Gl_3(\mathbf{F}_p)$.

7. Prove there is no simple group of order 637.