Midterm-MS403

October 22, 2013

(20)1. Complete the following definitions:

(a) A set G with a binary operation $*$ is called a group if

(b) A group G is called cyclic if

(c) Let G be a group and $g \in G$. The centralizer of g is

(d) Let V and W be vector spaces over the field F. A function $T : V \to W$ is called a linear transformation if

(20)2. Give examples of each of the following. No justification is required.

(a) A group G, $G \neq \{e\}$, for which $Z(G)$, the center of G, equals $\{e\}$.

(b) A proper subgroup of finite index in an infinite group.

(c) A noncyclic group of order 45.

(d) An element of order two in an infinite nonabelian group.

(10)3. State the fundamental theorem of homomorphisms for groups.

(10)4. (a) Find the exponent of S_6.

(b) Find the order of the centralizer of the following permutation in S_7.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 1 & 2 & 7 \end{pmatrix}$$

(10)5. Prove there is no homomorphism from D_4 onto C_4.

(12)6. True or False. No justification required.

(a) Every subgroup of a cyclic group is cyclic.

(b) If V and W are finite dimensional F-vector spaces of the same dimension, then V is isomorphic to W.

(c) Let V and W be finite dimensional F-vector spaces and $T : V \to W$ a linear transformation. If $dim(V) < dim(W)$, then T is one-to-one.

(d) In Q_8 every sugroup is normal.

(8)7. Find all of the elements in the cyclic group $(\mathbf{Z}_{24}, \oplus)$ that generate the whole group.

(10)8. Let V and W be finite dimensional vector spaces over the field F. Let $S : V \to W$ and $T : W \to V$ be linear transformations.

(a) Prove that if $dim(V) > dim(W)$, then the linear transformation $T \circ S : V \to V$ is not an isomorphism.

(b) Let A be an $m \times n$ matrix over F and B an $n \times m$ matrix over F. Prove that if $m > n$ then $AB \neq I$.