Additional exercise corrected

Let A be a subset of $\{1,2,\ldots,n\} \times \{1,\ldots,n\}$ that satisfies the following properties

(1) $(i,j) \in A \implies i \neq j$

(2) $(i,j) \in A \iff (j,i) \notin A$

(a) Show that if A' is another subset that satisfies (1) and (2), then there is a bijection from A to A'.

(b) Show that if f is a permutation of $\{1,2,\ldots,n\}$ then

$$\prod_{(i,j)\in A} \frac{x_{f(i)} - x_{f(j)}}{x_i - x_j} = \prod_{(i,j)\in A'} \frac{x_{f(i)} - x_{f(j)}}{x_i - x_j}$$

(c) Show that if g is another permutation

$$\prod_{i<j} \frac{x_{f(g(i))} - x_{f(g(j))}}{x_{g(i)} - x_{g(j)}} = \prod_{i<j} \frac{x_{f(i)} - x_{f(j)}}{x_i - x_j}$$