— M436 — Homework Assignment 5 —

Due: Monday, October 6, in class. Each problem is worth 20 points. Please show all your work. Homework that is illegible or discourages the reader otherwise from looking at it will be returned ungraded.

Exercise 1

This problem studies *ice cream geometry*, a special kind of incidence geometry. The points are a certain set of children in a class room, and the lines are ice-cream flavors. A child is incident with an ice-cream flavor if she likes that flavor.

The following axioms are known to be true:

- (A1) There are exactly five flavors of ice cream: vanilla, chocolate, strawberry, cookie dough, and bubble gum.
- (A2) Given any two different flavors, there is exactly one child who likes these two flavors.
- (A3) Every child likes exactly two different flavors among the five.

Investigate this geometry by answering the following questions:

1. How many children are there in this classroom? Prove your result.
2. Show that any pair of children likes at most one common flavor.
3. Show that for each flavor there are exactly four children who like that flavor.

Justify all your answers.

Exercise 2

Show that the Möbius transformations

$$f(z) = \frac{1}{1-z}$$
$$g(z) = \frac{z}{z-1}$$

generate a group of order 6 which is isomorphic to the symmetric group S_3 of three elements. Hint: What do these Möbius transformations do to 0, 1, and ∞?

Exercise 3

Let $\mathbf{FP}^2$ be a projective plane. A set of four points p_1, p_2, p_3, p_4 is a *projective frame* if no three of them are collinear. Suppose we have two projective frames p_1, p_2, p_3, p_4 and q_1, q_2, q_3, q_4. Show that there is a projective linear transformations that maps p_i to q_i for $i = 1, ..., 4$.

Exercise 4

Let $(G, \circ, 1)$ be a group. Define a relation $\sim$ on G by $a \sim b$ if and only if there is an element $g \in G$ such that $b = g \circ a \circ g^{-1}$. Show that this relation is an equivalence relation. For $G = S_4$ the symmetric group of four elements, determine the partition of S_4 into equivalence classes.

Exercise 5

In the projective plane $\mathbf{F}_3\mathbf{P}^2$, find a projective linear transformation in form of a 3×3 matrix that maps

$$(0 : 1 : 2) \mapsto (1 : 2 : 0) \mapsto (2 : 0 : 1) \mapsto (0 : 1 : 2)$$