
— M436 — Homework Assignment 6 —

Due: Wednesday, October 15, in class. Each problem is worth 20 points. Please show all your work. Homework that is illegible or discourages the reader otherwise from looking at it will be returned ungraded.

Exercise 1

Consider in \mathbf{RP}^2 the triangle with vertices $p_1 = (-3, -4)$, $p_2 = (3, -4)$, and $p_3 = (-3, 4)$. Determine the centers and radii of the circumcircle and incircle of this triangle. The point $q_1 = (3, 4)$ lies also on the circumcircle. Find the coordinates of the other two vertices q_2 and q_3 of the Poncelet triangle determined by q_1 and the two circles.

Exercise 2

Find a projective linear transformation of \mathbf{RP}^2 that maps the conic $x^2 + y^2 = z^2$ to the conic $xz = y^2$. Verify your claim.

Exercise 3

Find a conic in the projective plane $\mathbf{F}_5\mathbf{P}^2$ that contains the points $(1 : 1 : 2)$, $(1 : 1 : 3)$, $(1 : 3 : 0)$, $(1 : 4 : 2)$ and $(1 : 4 : 3)$. Find one more point on this conic.

Exercise 4

For which value of t is the conic $2x^2 - 2y^2 - tz^2 + 3xy - xz + 3yz = 0$ in \mathbf{RP}^2 degenerate, and in which two lines does it decompose in this case?

Exercise 5

In \mathbf{RP}^2 , find the two tangents to the conic $3x^2 - y^2 + z^2 + 2xz = 0$ that pass through the point $(1 : 1 : -2)$. Find also the points where these lines touch the conic.