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Exam 1

Justify your answers!
Numerical expressions may be left unsimplified.

1. [12 pt.s] A population model with 3 age cohorts has the growth matrix

$$A = \begin{bmatrix} 0 & 2 & 5 \\ 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix}$$

which has the following eigenvalues:

$$\begin{gathered} -0.302252 + 0.272156\,\mathbf{i} \\ -0.302252 - 0.272156\,\mathbf{i} \\ 0.604504 \end{gathered}$$

and the following eigenvectors:

$$\begin{bmatrix} 0.964455 \\ -0.176218 - 0.158671\,\mathbf{i} \\ 0.0121855 + 0.115965\,\mathbf{i} \end{bmatrix}, \begin{bmatrix} 0.964455 \\ -0.176218 + 0.158671\,\mathbf{i} \\ 0.0121855 - 0.115965\,\mathbf{i} \end{bmatrix}, \begin{bmatrix} -0.985157 \\ -0.162969 \\ -0.0539184 \end{bmatrix}$$

(a) What is the long term growth rate of this population?

(b) What is the long term age distribution of the population?

(c) Suppose the fecundity rates for the second and third age cohorts are replaced by $2a$ and $5a$ respectively, where a is a real constant. What is the minimum value of a that would allow the population to survive in the long term?

2. [6 pt.s] Suppose another population with four age cohorts has growth matrix

$$A = \begin{bmatrix} 0 & 10 & 10 & 0 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \end{bmatrix}.$$

If you could increase one of the nonzero values in that matrix by 20%, i.e. multiply it by 1.2, which one would you choose to change in order to maximize the growth rate?

3. [12 pt.s] Suppose a small local business has only one clerk working at a time to ring up sales for customers. During the busy part of the day they expect an average of 20 customers per hour who make purchases, while the clerk requires an average of 2 minutes to ring up each purchase.

(a) What is the probability that the clerk is idle at a random moment?

(b) What is the probability that there is no checkout line, i.e. that there is no more than one customer wanting to make a purchase?

(c) How much time on average does a customer spend waiting in line for the clerk?

(d) If the average number of buying customers per hour increased by 10%, by how much would the clerk need to reduce his average time per customer (i.e., increase his efficiency) so that the average time spent waiting in line remains the same?