M463 Homework 15

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Suppose X with values $(0,1)$ has density $f(x) = cx^2(1-x)^2$ for $0 < x < 1$. Find:

a) the constant c;

Solution: The density function must satisfy:

$$\int_0^1 f(x)dx = 1 \Rightarrow \int_0^1 cx^2(1-x)^2 dx = 1 \Rightarrow c\int_0^1 x^2 - 2x^3 + x^4 dx = 1 \Rightarrow c\left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right]_0^1 = 1 \Rightarrow \boxed{c = 30}$$

b) $E(X)$;

Solution: By definition:

$$E(X) = \int_0^1 xf(x)dx = \int_0^1 x30x^2(1-x)^2 dx = 30\int_0^1 x^3 - 2x^4 + x^5 dx = 30\left[\frac{x^4}{4} - 2\frac{x^5}{5} + \frac{x^6}{6}\right]_0^1 = 30\left[\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right] = \boxed{\frac{1}{2}}$$

c) $Var(X)$;

Solution: First compute the second moment of X:

$$E(X^2) = \int_0^1 x^2 f(x)dx = \int_0^1 x^2 30x^2(1-x)^2 dx = 30\int_0^1 x^4 - 2x^5 + x^6 dx = 30\left[\frac{x^5}{5} - \frac{x^6}{3} + \frac{x^7}{7}\right]_0^1 = 30\left[\frac{1}{5} - \frac{1}{3} + \frac{1}{7}\right] = \boxed{\frac{2}{7}}$$

Hence,

$$Var(X) = E(X^2) - E(X)^2 = \frac{2}{7} - \frac{1}{4} = \boxed{\frac{1}{28}}$$