Name: **M463**
July 1, 2005 **Exam 1**

All six problems are worth 10 points each.

1. Suppose 4 ordinary dice are rolled. What is the chance that at least one number appears more than once?

2. A box contains one black ball and one white ball. A ball is drawn at random, then replaced in the box with an additional ball of the *opposite* color. Then a second ball is drawn from the three balls now in the box. What is the probability that the first ball drawn was white, given that at least one the two balls drawn was *black*?

3. Cards are dealt from a shuffled, standard deck until the first red card appears. For the following you may leave your answers in unsimplified form. What is the probability that 3 or fewer deals are required?

4. In professional women's tennis, a match is won by the winner of 2 out of three sets. Suppose that two players, A and B, play a match, and player A has probability p of winning an individual set.

   What is the probability (in terms of p and $q = 1 - p$) that player A wins the match?

5. A hat contains 2 coins, both of which are unfair; one land heads with probabililty $\frac{1}{3}$, and the other with probability $\frac{2}{3}$. A coin is chosen from the hat at random, tossed, and lands heads. Then it is flipped again. What is the probability the result of the second flip is also heads?

6. Two players, Abner and Barbara, play a simple game with an urn containing 9 white and 1 red ball. They take turns drawing a ball from the urn, and the first to draw the red ball wins. If a black ball is drawn it is replaced in the urn before the next player's turn. If Abner goes first, what is the probability he wins the game?

(1) $\Omega = \{(d_1,d_2,d_3,d_4) : d_i \text{ is result from die } i\}$. $\Rightarrow |\Omega| = 6^4$

P(at least one number appears more than once) = 1 − P(all distinct numbers)

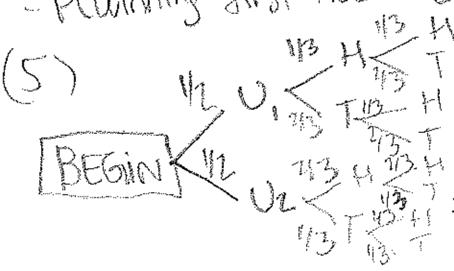
$= 1 - \frac{(6\cdot5\cdot4\cdot3)}{6^4} = 1 - \frac{60}{216} = \frac{216-60}{216} = \frac{156}{216} = \frac{78}{108} = \frac{39}{54} = \boxed{\frac{13}{18}}$ ✓

(2) Already did in quiz week 1. Ans: $\boxed{\frac{2}{5}}$ ✓

(3) P(3 or fewer deals) = 1 − P(4 or more) = 1 − P(first 3 are black) $= \boxed{1 - \frac{26}{52}\cdot\frac{25}{51}\cdot\frac{24}{50}}$ ✓

(4) P(A winning an individual set) = p. P(A winning match) = P(A winning 2 out of 3)
= P(winning first two or winning first and last or winning last two) $= p^2 + p^2q + p^2q$
$= \boxed{p^2(1+2q)}$ ✓

(5)



$P(HH \mid \text{first toss } H) = \frac{P(HH \cap \text{first } H)}{P(\text{first } H)} = \frac{P(HH)}{P(\text{first } H)}$

$= \frac{P(\{U_1HH, U_2HH\})}{P(\{U_1HH, U_1HT, U_2HH, U_2HT\})} = \frac{\frac{1}{18}+\frac{4}{18}}{\frac{1}{18}+\frac{4}{18}+\frac{2}{18}+\frac{2}{18}} = \frac{\frac{5}{18}}{\frac{9}{18}} = \boxed{\frac{5}{9}}$ ✓

urn = 9w , 1r ; with replacement.

Abners goes first.

P(Abners winning) = P(r OR wwr OR wwwwr OR wwwwwwr OR ... even # 9w r)

these are all disjoints events; Hence:

$$= \frac{1}{10} + \frac{9}{10}\cdot\frac{9}{10}\cdot\frac{1}{10} + \frac{9}{10}\cdot\frac{9}{10}\cdot\frac{9}{10}\cdot\frac{9}{10}\cdot\frac{1}{10} + \cdots$$

$$= \frac{1}{10} + \left(\frac{9}{10}\right)^2\cdot\frac{1}{10} + \left(\frac{9}{10}\right)^4 + \frac{1}{10} + \left(\frac{9}{10}\right)^8 \frac{1}{10} + \cdots$$

$$= \frac{1}{10}\left(1 + \left(\frac{9}{10}\right)^2 + \left(\frac{9}{10}\right)^4 + \left(\frac{9}{10}\right)^8 + \cdots\right)$$

$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}$

$$= \frac{1}{10}\left(1 + \left[\left(\frac{9}{10}\right)^2\right]^1 + \left[\left(\frac{9}{10}\right)^2\right]^2 + \left[\left(\frac{9}{10}\right)^2\right]^3 + \cdots\right]$$

this is a geometric series, it converges since $|r| < 1$

$$= \frac{1}{10}\left[\frac{1}{1-\left(\frac{9}{10}\right)^2}\right] = \frac{1}{10}\left[\frac{1}{1-\frac{81}{100}}\right] = \frac{1}{10}\left[\frac{1}{\frac{100-81}{100}}\right] = \frac{1}{\cancel{10}}\left[\frac{10\cancel{0}}{19}\right] = \boxed{\frac{10}{19}}$$

Geometric Distribution:

$$P(X=k) = q^{k-1}p.$$

$$P(\text{Abners wins}) = p + q^2p + q^4p + \cdots = p(1 + q^2 + q^4 + \cdots) = \frac{1}{1+q} = \frac{1}{1+\frac{9}{10}} = \frac{1}{\frac{19}{10}} = \boxed{\frac{10}{19}}$$

$x = P(\text{Abners win})$

$y = P(\text{Barbara wins})$

$x + y = 1$

$x = p + q^2 x$

P(Abners win | no one won on first two turns) = P(Abners win).

$x - q^2 x = p$

$$x = \frac{p}{1-q^2}$$