Fostering Learning Outside the Classroom: Homework

In the mathematics classroom, students are exposed to new concepts, definitions and techniques. Even the brightest students, though, cannot develop a lasting understanding of the material presented in real time. In the university setting, the learning occurs when students work problems outside the classroom. When I design problem sets for the textbooks I have written, I try to design problems with four different goals in mind:

- 1. Problems that help students learn the basic definitions and vocabulary.
- 2. Practice problems that focus on developing procedural memory.
- 3. Higher-order thinking problems that extend and abstract basic concepts and procedures or approach them from a different angle to stimulate deeper understanding.
- 4. Complex problems that integrate several concepts and or procedures and require problem-solving skills.

Most good textbooks include all of these types of problems. The number of each type of problem you assign depends on your students. I have found that slower math learners need to work many more problems than the more talented students if they are to develop a lasting procedural memory or understanding. For example, I think that the problems in Maki-Thompson are entirely appropriate for the average M118 student, but there are not enough type 1 and 2 problems available for the slower math learners in D116. Instructors for D116 and D117 often supplement the textbook problems with more basic practice problems so that their students are better able to work their way up to the type 3 and 4 problems in the text.

Ideally, when students do their assignments, they would progress through problems in a logical order, developing skill, understanding and (hopefully) confidence as they go. Some instructors tend to focus only on the end result rather than the process, and expect students to do only a few of the hardest problems per set. Others are careful to avoid any redundancy in their problem choices. I think it is much more effective to ask students to do a wide range of problems, including some sets of similar problems. For example, when teaching integration techniques (integration by substitution, parts, partial fractions, trig substitution, etc.) some instructors assign only one or two problems each of type. Although this may be enough practice to learn how to use each method, it is certainly not enough practice to help students learn when to use each technique.

When I assign problems, I include as many problems as *I think* my students will need to develop the level of proficiency that their exams will require. This is almost certainly more problems than they have been asked to do in high school, but not so

many that hardworking students will refuse to do them. My students work a lot of problems, but they also frequently make a better showing on the departmental final than students who have been shorter assignments. In all of the classes I coordinate, it is also clear that working lots of problems is a *necessary* condition for doing well in the class. (Unfortunately, if is not always a sufficient condition, but that is a topic for another day.)

In the low-level classes I coordinate, the amount of homework I assign frequently seems excessive to both students and the AIs who teach them. Certainly the average mathematics graduate student did not require that much homework to learn basic algebra. However, instructors of low-level math classes cannot assume that their students are able to learn in the same way or at the same rate that they did. Some students really do need to put in more time and effort to develop a lasting understanding of mathematics.

Most courses taught by associate instructors are taught in reasonably small sections. My preference when teaching a class with 80 or fewer students is to assign written assignments and do much of my own grading. When I teach calculus, I generally collect homework once a week, and select 10 problems to grade from the week's assignments. I think that it is very important for instructors see their students work. By grading even a few problems, you are able to see the types of mistakes students make and develop a much better idea of what they do not understand. You also have the opportunity to require students to use "good form" when doing their homework. Many students use too few steps, omit parentheses, and use bad notation which is likely to produce errors.