S520 Homework 7

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10.5.#A-1:

(a) 1-sample t-test.

$$t_n = \frac{\bar{x_n}-\mu_0}{s_n/\sqrt{n}} = \frac{3.194887-0}{\sqrt{104.0118}/\sqrt{400}} = \frac{3.194887}{10.19862/20} = \frac{3.194887}{0.509931} = 6.265332$$

(b) Expresion $iv. = 1 - pt(1.253067, df = 399)$, best approximates the significance probability.

(c) True: $\mathbf{p} = 0.03044555 < 0.05 = \alpha \Rightarrow$ reject the null hypothesis.

10.5.#C:

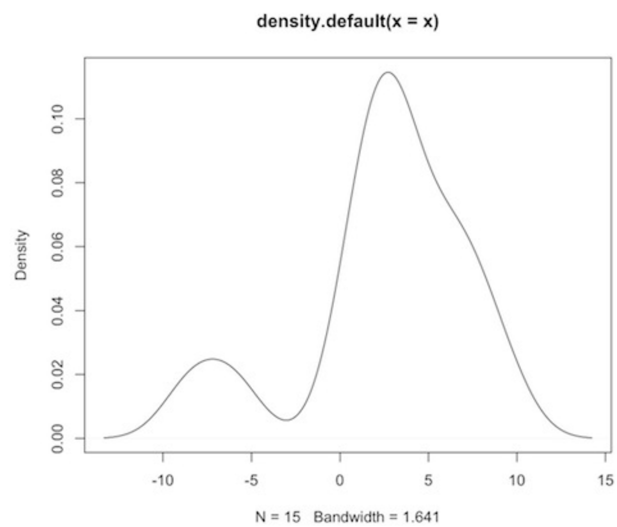
1-(a) The experimental unit is a pair (cross,self) of the same age. The measurement taken on each unit is (cross height, self height), where height is in inches. For short, let Hc_i be the height of the cross-fertilization i and Hs_i be the height of the self-fertilization i.

1-(b) Let $X_i = Hc_i - Hs_i$. Then $X_1, X_2, ..., X_{15} \sim P$

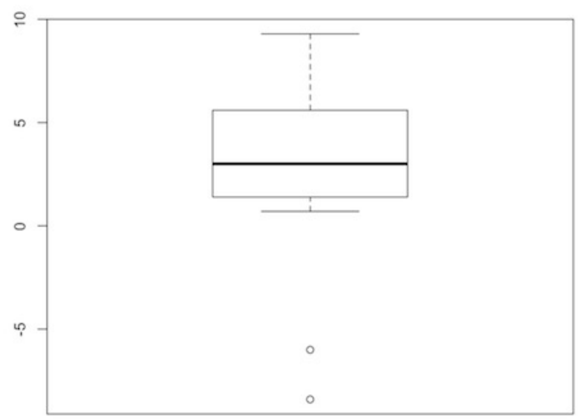
1-(c) $H_0 : \theta \leq 0$ vs $H_1 : \theta > 0$

2-(a) Let $\vec{x} = x = c(6.1, -8.4, 1, 2, 0.7, 2.9, 3.5, 5.1, 1.8, 3.6, 7.0, 3.0, 9.3, 7.5, -6.0)$. Then,

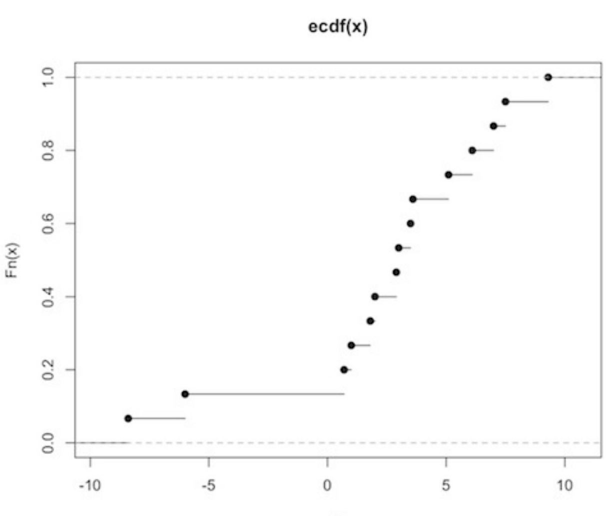
(a) $plot(density(x))$



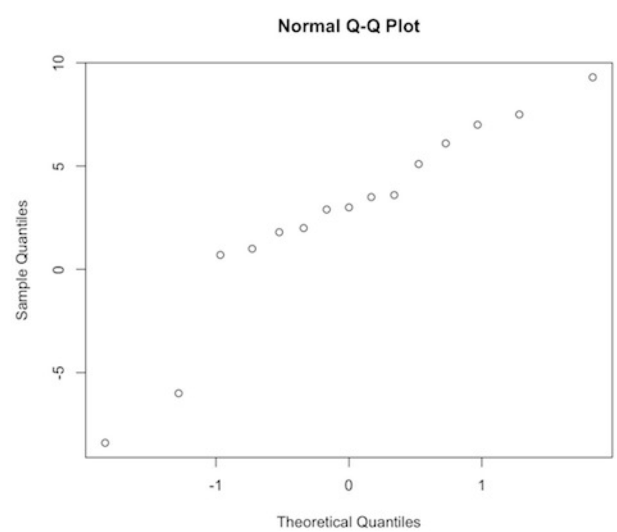
(e)$boxplot(x)$:



(f) $plot(ecdf(x))$:



(g)$qqnorm(x)$:



We can also calculate $iqrx/sqrt(var(x)) = 0.8911863$. From this observations and graphs, there seems to be evidence of normality to this data set. At first, the iqr to stdev ratio seems a little off, but by doing some experiments with R with a normal distribution of size 15, one can confirm that the value of this ratio is feasible for a sample of size 15. Therefore, if we are not being too conservative, we can conclude that this data is normally distributed.

2-(b) Assuming it is normally distributed, which we can as stated before, then we can assume symmetry.

3-(a) First, we need to estimate σ^2, i.e., $Sn^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2 = 22.2107$.
Second, the test statistics, i.e., $t_n = \frac{\bar{x}_{15}-\mu_0}{s_n/\sqrt{n}} = \frac{2.6067-0}{\sqrt{22.2107}/\sqrt{15}} = \frac{2.6067}{4.712823/3.872983} = \frac{2.6067}{1.216846} = 2.1422$.
Finally, we can calculate the significance probability, i.e., $1 - pt(2.1422, 14) = 0.0251$.
Because $\mathbf{p} = 0.0251 \leq 0.05 = \alpha$, then we reject H_0.

3-(b) $1 - \alpha = 0.90 \Rightarrow \alpha = 0.10$. Then, $q_t = q_t(.95, df = 14) = 1.7613$. The confidence interval is

$$(\bar{x} - q_t\frac{s_n}{\sqrt{n}}, \bar{x} + q_t\frac{s_n}{\sqrt{n}}) = (2.6067 - 1.7613\frac{4.7128}{\sqrt{15}}, \bar{x} + 1.7613\frac{4.7128}{\sqrt{15}}) = (0.4634, 4.7499)$$

10.5.#D:

1. Both the ratios and the log of the ratios are very similar when tested for normality. However, the log of the ratios behave a little more like a normal distribution. This conclusion can be made by looking at the box plot, density, qqnorm and cumulative distribution function. In all cases the log of the ratios is slighter better distributed with respect to a normal distribution, typically more shifted to the right. In particular, the density plot of the ratios has two bumps to the right whereas the density of the log of the ratios only has one. Finally, the iqr to stdev for each case is: (i) for the ratios $IQR(rec)/sqrt(var(rec)) = 0.7620725$ and (ii) for the log $IQR(reclog)/sqrt(var(reclog)) = 0.8544223$, and thus, the log ratio is closer to what we would expect from a normal distribution.

2. I would use the **log of the ratios** for which an assumption of normality seems more plausible. Therefore the mean we would like to test now is $log(0.618034) = -0.4812$.
For the hypothesis testing, from the point of view of the anthropologist, would be:

$$H_0 : \mu = -0.4812. \text{ vs. } H_1 : \mu \neq -0.4812$$

One could argue that the anthropologist wants to minimize Type I error, i.e., that the Shoshoni civilization actually used golden rectangles but the test shows otherwise. This is why in the test H_0 represent the golden ratio.
To calculate the Student's 1-sample t-test,we need the mean, i.e., $\bar{x}_{20} = -0.4231$ and we need to approximate σ^2 i.e., compute $Sn^2 = 0.0166$, therefore $Sn = \sqrt{0.0166} = 0.1288$

$$t_n = \frac{\bar{x}_{20} - \mu_0}{s_n/\sqrt{n}} = \frac{-0.4231 + 0.4812}{0.1288/\sqrt{20}} = \frac{0.0581}{0.0288} = 2.02$$

Therefore,

$$\mathbf{p} = 2 * pt(-2.02, df = 19) = 0.05771 > 0.05 = \alpha \Rightarrow \text{ fail to reject } H_0$$