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STAT-S 420/620, Spring 2014: Test 1

Name: Enrique Areyan Circle one: **420** or **620**

Rules: You have 75 minutes to complete the test. You are permitted the use of a formula sheet and a calculator. Please write your answers on the back of this page and/or on the additional sheet of paper provided. *You must show your work to obtain full credit.*

1. Let $\theta = 1, 2, 3$ index the following three probability distributions for generating $x \in \mathcal{X} = \{1, 2, 3, 4, 5\}$:

	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$
P_1	0.50	0.35	0.05	0.05	0.05
P_2	0.25	0.20	0.10	0.20	0.25
P_3	0.10	0.20	0.40	0.20	0.10

5 (a) (10 points) Let $\alpha = 0.10$. Is there a uniformly most powerful test of $H_0 : \theta \in \{1\}$ versus $H_1 : \theta \in \{2, 3\}$? Why or why not?

10 (b) (10 points) Suppose that a prior distribution π assigns probability 0.50 to $\theta = 1$, probability 0.25 to $\theta = 2$, and probability 0.25 to $\theta = 3$. What is the conditional (posterior) distribution of $\theta | x = 3$?

5 (c) (10 points) Suppose that we desire to take action $a \in \mathcal{A} = (-\infty, \infty)$ with loss function $L(\theta, a) = (\theta - a)^2$. Let d_π denote the Bayes rule for taking this action. What action should we take if we observe $x = 3$, i.e., what is $d_\pi(3)$?

27 2. Let $\theta = 1, 2$ index two possible states of nature and suppose that a decision problem has six nonrandomized decision rules with the following risk functions:

	d_1	d_2	d_3	d_4	d_5	d_6
$R(\theta, d_i)$	(0, 5)	(1, 2)	(6, 0)	(5, 6)	(4, 7)	(2, 7)

(a) (10 points) Using the graph paper provided with this test, sketch the risk set for $\mathcal{D}$. the collection of all randomized decision rules.

(b) (10 points) Identify the admissible rules in $\mathcal{D}$.

(c) (10 points) Which rule is minimax in $\mathcal{D}$?

$$\int_{-\infty}^{\infty} R(\theta, d)\, \pi(\theta)\, d\theta$$