INFO I201
Homework 6

Due 05/30.

- **Reading assignment**: Chapter 3 of the book.
- **Regular problems**:

1. Give natural deduction proofs for the following sequents:

   (a) $\neg A \vee B \vdash A \longrightarrow B$

   (b) $(A \wedge B) \vee C \vdash C \vee B$

   (c) $\vdash P \vee \neg P$

   (d) $\neg(A \wedge B) \vdash \neg A \vee \neg B$

   (e) $A \longrightarrow C, B \longrightarrow C, B \longrightarrow D \vdash (A \wedge B) \longrightarrow (C \wedge D)$

   (f) $P \longrightarrow R, Q \longrightarrow R \vdash (P \vee Q) \longrightarrow R$

2. Suppose that $A \times B = \emptyset$, where A and B are sets. What can you conclude? Explain.

3. Let $A = \{a, b, c\}$, $B = \{x, y\}$ and $C = \{0, 1\}$. Find

   - $A \times B \times C$
   - $C \times B \times A$
   - $C \times A \times B$
   - $B \times B \times B$

4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

   - $A \cup B$
   - $A \cap B$
   - $A - B$
   - $B - A$

5. Let A and B be sets. Show that,

   - $(A \cap B) \subseteq A$
   - $A - B \subseteq A$
   - $A \subseteq (A \cup B)$
   - $A \cup (B - A) \subseteq A \cup B$

6. Suppose $A \cap C = B \cap C$ for some sets A, B and C, can we conclude that $A = B$?

7. Suppose $A \cup C = B \cup C$ for some sets A, B and C, can we conclude that $A = B$?