INFO I201
Homework 8
Due 06/06

- **Reading assignment**: Sections 4.1-4.3.
- **Regular problems**:

1. Consider the following language: $\mathcal{L}$: Constants: m, Predicate Symbols: $B(x,y), S(x,y)$, Function Symbols: $f(x)$.

   Decide if each expression below is a formula:

   - $S(m,x)$
   - $\forall x \, \exists y S(x, f(y))$
   - $B(m, f(m))$
   - $f(m)$
   - $B(B(m,x),y)$
   - $(B(x,y) \longrightarrow (\exists z S(z,y)))$
   - $B(f(f(x)), S(m,x))$

2. Consider the language we discussed in class, namely the language $\mathcal{L}$: Constants: P, J, Predicate Symbols: $H(x,y), L(x,y)$, Function Symbols: $F(x)$.

   Decide if each expression is a term:

   - $H(x, L(x,y))$
   - $F(F(F(x)))$
   - $L(F(x), F(F(P)))$
   - P

   Decide if each of the following expressions is a formula:

   - $P \wedge J$
   - $\forall x L(x, F(y))$
   - $L(P, H(P,J))$
   - $F(L(x,P)) \wedge F(J)$

3. Let $P(x,y)$ be the statement "student x has taken class y". Express each of the formulas below in English:

   (a) $\exists x \exists y P(x,y)$
   (b) $\exists x \forall y P(x,y)$
   (c) $\forall x \exists y P(x,y)$

(d) $\exists y \forall x P(x, y)$

(e) $\forall x \forall y P(x, y)$

4. Let $C(x, y)$ mean "student x is enrolled in class y". Express each of the formulas below in simple English:

(a) $C(Randy, CS201)$

(b) $\exists y C(Carol, y)$

(c) $\exists x (C(x, M222) \wedge C(x, I201))$

5. Consider the language $\mathcal{L}$:

Constant Symbols: $0, 1$
Predicate Symbols: $L(x, y)$
Function Symbols: $p(x, y), m(x, y)$

Are the following terms in this language?

(a) $p(0, 0)$

(b) $p(0, m(1, 0))$

(c) $p(p(1, 1), m(p(1, 1), p(1, 1)))$

(d) $m(m(m(1, 1), 1), 1)$

Now suppose that $U = \mathbb{Z}$, $0, 1$ are the numbers zero and one respectively and that $p(x, y) = x + y$, and $m(x, y) = x \times y$, and that $L(x, y)$ means that $x < y$. What numbers do the terms above refer to?

Find a way of saying that $3 < 4$ in this language.

6. Consider the language $\mathcal{L}_1$ defined as:

Predicate Symbols: $L(x, y), H(x, y)$
Function Symbols: $f(x)$.
Consider the model $M = (\{a, b, c, d\}, I)$ where $I(H) = \{(a, b), (b, c), (a, c)\}, I(L) = \{(c, c), (c, b), (c, a)\}$ and $I(f)(a) = b, I(f)(b) = a, I(f)(c) = c$ and $I(f)(d) = d$. Let $\rho(x) = a$ and $\rho(y) = d$. Determine the truth value of each formula below in the model M and environment ρ.

(a) $L(x, x) \vee H(x, y)$

(b) $H(x, y)$

(c) $H(f(x), f(y))$

(d) $L(f(x), x)$

(e) $\forall x \exists y H(x, y)$

(f) $\exists x H(x, y)$

(g) $\forall x \forall y H(x, y)$

(h) $\forall x \forall y L(x, y)$