INFO I201
Final Exam

Thursday, June 13, 2013
Duration: 100 minutes.

Q1. (6 pts.) Give natural deduction proofs for the following sequents:

(i) $Q \longrightarrow R \vdash (P \vee Q) \longrightarrow (P \vee R)$

(ii) $A \longrightarrow (B \longrightarrow C) \vdash (A \longrightarrow B) \longrightarrow (A \longrightarrow C)$

Q2. (5 pts.) Consider the following argument:

> Either the game will not be played and it will not be broadcast, or it will not be played and not rescheduled. It stands to reason that if there is a public outcry about the cancellation, and enough people write letters of complaints to the managing officials, then the game will be played. So, either there will be no public outcry about the cancellation, or not enough people will write letters of complaint to the managing officials.

Translate the statements above into propositional logic using propositional letters: P, B, R, O, L. Determine if the argument is valid or not. Provide sufficient explanation.

Q3. (4 pts.) Let $\mathcal{L}$ be a language with two constant symbols F and D and one binary predicate symbol W. Define a model $M = (\{Fred, Mary, David, Jane\}, I)$ such that all the formulas below are true in this model. Note that $I(F) = Fred$ and $I(D) = David$.

(i) $\exists x\, W(F, x)$

~~(ii) $\exists x\, W(x, D)$~~

(iii) $W(D, F) \wedge \forall x\, W(F, x)$

(iv) $\forall x\, (W(D, x) \longrightarrow W(F, x))$

Q4. (5 pts.) Consider the first order language $\mathcal{L}$ with two unary predicate symbols A and B.

(i) Consider the model $M = (U, I)$ where $U = \{a, b, c\}$, $I(A) = \{a\}$ and $I(B) = \{a, c\}$. Is the formula

$$\psi \equiv \forall x(A(x) \longrightarrow B(x)) \longrightarrow \forall x((A(x) \vee B(x)) \leftrightarrow A(x))$$

true in this model? Explain your answer completely.

(ii) Design a model (i.e. define $I(A)$ and $I(B)$) such that with the same universe $U = \{a, b, c\}$, the formula

$$\phi \equiv \forall x(A(x) \vee B(x)) \longrightarrow (\forall x A(x) \vee \forall x B(x))$$

is false.

Q5. (5 pts.) (i) Prove or disprove: If $A \neq \emptyset$ and $A \in \mathcal{P}(B)$, then $A \cap B \neq \emptyset$.

(ii) Prove or disprove: If $A \cap B \neq \emptyset$, then $A \cup B \subseteq A$.

Q6. (5 pts.) Consider the function $f : \mathbb{N} \longrightarrow \mathbb{N}$ defined by $f(n) = n^2$. Also, let $E \subseteq \mathbb{N}$ be the set of even numbers and $O \subseteq \mathbb{N}$ be the set of odd numbers. Define $f(E) = \{f(n) \mid n \in E\}$, and $f(O) = \{f(n) \mid n \in O\}$, and $f(\mathbb{N}) = \{f(n) \mid n \in \mathbb{N}\}$.

(i) What is $f(\mathbb{N}) \cap \{0, 1, \cdots, 100\}$?

(ii) Is $f(\mathbb{N}) = \mathbb{N}$? Explain.

(iii) Find $f(E) \cap f(O)$.

(iv) Is $f(\mathbb{N}) - \mathbb{N} = \emptyset$? Explain.

(v) Find $f(O) \cap E$.

Q7. (6 pts.) **Extra Credit.** Let $\mathcal{L}$ be a language with a binary predicate symbol $R(x, y)$. Also consider the model $M = (U, I)$ where $U = \{a, b, c, d\}$ and $I(R) = \{(a, a), (b, b), (b, c), (a, c), (c, b)\}$.

(i) Consider the formula $\phi_1 = \forall x \forall y[(R(x, y) \wedge R(y, x)) \longrightarrow x = y]$. Is ϕ_1 valid in the model M?

(ii) Consider the formula $\phi_2 = \exists x[R(x, x) \wedge \forall y(R(y, y) \longrightarrow (y = x))]$. Find a model $M' = (\{a, b, c, d\}, I')$ (i.e., define $I'(R)$) such that ϕ_2 is true in the model M'.

(iii) Consider the formula $\phi_3 = \forall x R(x, x) \wedge \forall x \forall y[R(x, y) \longrightarrow R(y, x)]$. Find a model $M'' = (\{a, b, c, d\}, I'')$ (i.e., define $I''(R)$) such that ϕ_3 is true in the model M''.