INFO I201
Sample Final Exam

Not to be collected.

Q1. Give natural deduction proofs for the following sequents:

(i) $\neg(A \vee B) \vdash \neg A \wedge \neg B$

(ii) $(A \vee B) \longrightarrow C \vdash (A \longrightarrow C) \wedge (B \longrightarrow C)$

Q2. Consider the following argument :

> No one denies that the earth is flat. Now, either we eventually sail off the end and are drowned in a pit of demons, or we are taken up by angels just before plunging headlong into the abyss. Either there is no end to the earth, or else the earth is not flat, or for other unknown reasons we cannot ever sail off the end. From this, we must conclude that if there is an end to the earth, then we shall be taken up by angels just before plunging headlong into the abyss.

Translate this argument into the language of propositional logic using the following propositional letters:

- F = The earth is flat.
- S = We eventually sail off the end.
- D = We are drowned in a pit of demons.
- A = We are taken up by angels just before plunging headlong into the abyss.
- E = There is an end to the earth.

Either prove that the argument is valid or give a counterexample to its validity.

Q3. Consider the first order language $\mathcal{L}$ with two unary predicate symbols A and B.

(i) Consider the model $M = (U, I)$ where $U = \{a, b, c\}$, $I(A) = \{a\}$ and $I(B) = \{a, c\}$. Is the formula

$$\psi = \forall x[A(x) \longrightarrow B(x)] \longrightarrow \forall x[(B(x) \wedge A(x)) \longleftrightarrow A(x)]$$

true in this model? Explain your answer completely.

(ii) Give an interpretation (i.e. determine $I(A)$ and $I(B)$) such that with the same universe $U = \{a, b, c\}$, the formula

$$\phi = [\exists x A(x) \wedge \exists x B(x)] \longrightarrow \exists x[A(x) \wedge B(x)]$$

is false.

Q4.

- Prove or disprove: If $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.
- Prove or disprove: If $A \subseteq B$, then $A \cup B \subseteq A$.

Q5. Explain your answers **completely** in the following: Consider the function $f : \mathbb{N} \longrightarrow \mathbb{N}$ defined by $f(n) = n^2$. Let $E \subseteq \mathbb{N}$ be the set of even numbers and $O \subseteq \mathbb{N}$ be the set of odd numbers. Define $f(E) = \{f(n) \mid n \in E\}$, and $f(\mathbb{N}) = \{f(n) \mid n \in \mathbb{N}\}$.

(i) Find $f(E) \cap O$.

(ii) Is $f(\mathbb{N}) - \mathbb{N} = \emptyset$?

(iii) Is $\mathbb{N} - f(\mathbb{N}) = \emptyset$?

Q6. Let $\mathcal{L}$ be a language with a binary predicate symbol $R(x, y)$. Also consider the model $M = (U, I)$ where $U = \{a, b, c, d\}$ and $I(R) = \{(a, b), (b, c), (b, a), (a, c), (a, a), (b, b)\}$.

- Consider the formula $\phi_1 = \forall x \forall y \forall z[(R(x, y) \wedge R(y, z)) \longrightarrow R(x, z)]$. Is ϕ_1 valid in the model M?
- Consider the formula $\phi_2 = \forall x \forall y[R(x, y) \longrightarrow R(y, x)]$. Is ϕ_2 valid in the model M?
- Consider the formula $\phi_3 = \exists x \forall y[R(x, x) \wedge ((y \neq x) \longrightarrow R(x, y)]$. Give a model $M' = (\{a, b, c, d\}, I')$ (i.e., define $I'(R)$) such that ϕ_3 is true in the model M'.