Whole numbers and Operations
Activity 1
Math-T101 Spring 2014

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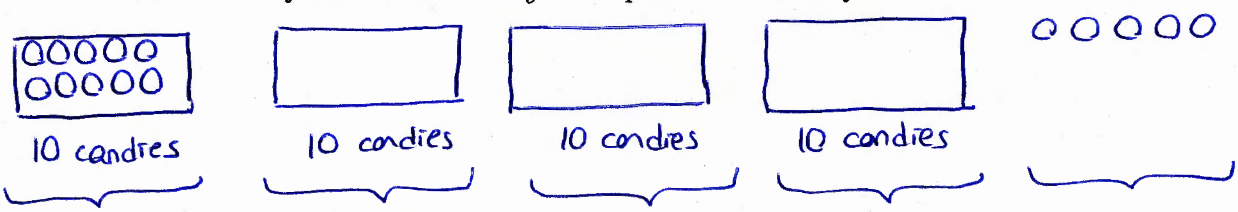
Problem 1. *Suppose candies are put in bags of 10. Draw a picture to show how many bags you could make with 45 candies and how many candies you would have left over. According to this picture, express the number of candies using multiplication and addition and state what each of the numbers in your expression stands for.*

type of measurement division which will be learned later



1 bag of candies + 1 bag of candies + 1 bag of candies + 1 bag of candies + 5 candies leftover

= 4 full bags of candies + 5 candies leftover

$(4 \times 10) + 5 = 45$

4 = # of bags; 10 = # of candies in each bag; 5 = leftover candies; 45 = # of total candies

Problem 2.

(a) How many bags of 10 candies can you make with 317 candies and how many candies would you have left over? Based on your answer, express the number of candies using multiplication and addition and state what each of the numbers in your expression stands for.

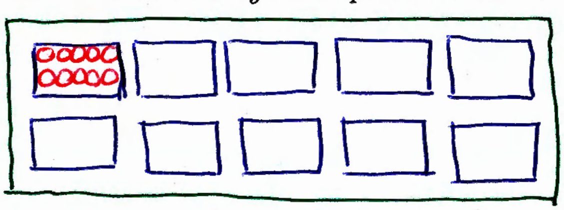
$$\begin{array}{r} 31 \\ 10\,\overline{)\,317} \\ -30 \\ \hline 17 \\ -10 \\ \hline 7 \end{array}$$

$(31 \times 10) + 7 = 310 + 7 = 317$

31 = # of bags; 10 = # of candies in each bag; 7 = leftover candies; 317 = total number of candies

31 bags of candies and 7 leftover

(b) Suppose that bags of candy are put into boxes, with 10 bags in each box. How many boxes, left-over bags, and left-over candies would you have? Based on your answer, express the total number of candies again, using multiplication and addition and state what each of the numbers in your expression stands for.



1 bag = 10 candies

1 box = 10 bags = 100 candies

$$\begin{array}{r} 3 \\ 10\,\overline{)\,31} \\ -30 \\ \hline 1 \end{array}$$

⇒ 31 bags of candies = 3 boxes of 10 bags in each and 1 lefover bag

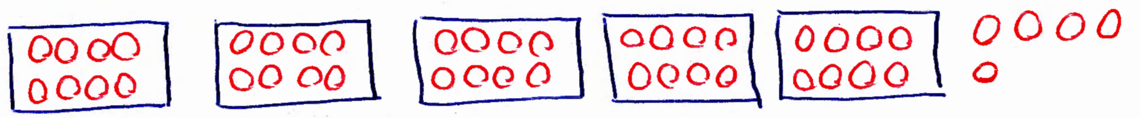
$$\begin{array}{r} 3 \\ 100\,\overline{)\,317} \\ -300 \\ \hline 17 \end{array}$$

⇒ 317 candies = 3 boxes of candies and 1 bag of candies leftover and 7 candies leftover

$(5 \times 8) + 5 = 45$

# of bags — # of candies in each bag — # of leftover candies — total # of candies

Problem 3. *Suppose candies are put in bags of 8. Draw a picture to show how many bags you could make with 45 candies and how many candies you would have left over. According to this picture, express the number of candies using multiplication and addition and state what each of the numbers in your expression stands for.*



8 candies

1 bag of candies + 1 bag of candies + 1 bag of candies + 1 bag of candies + 1 bag of candies + 5 candies leftover = 5 full bags of candies and 5 candies leftover

Problem 4. $\boxed{(5 \times 8) + 5 = 45}$

(a) How many bags of 8 candies can you make with 317 candies and how many candies would you have left over? Based on your answer, express the number of candies using multiplication and addition and state what each of the numbers in your expression stands for.

$$\begin{array}{r} 39 \\ 8\,\overline{)\,317} \\ -24 \\ \hline 77 \\ -72 \\ \hline 5 \end{array}$$

39 bags of 8 candies in each & 5 candies leftover

$(39) \times (8) + 5 = 317$

# of bags — # of candies in each bag — # of leftover candies — total # of candies

(b) Suppose that bags of candy are put into boxes, with 8 bags in each box. How many boxes, left-over bags, and left-over candies would you have? Based on your answer, express the total number of candies again, using multiplication and addition and state what each of the numbers in your expression stands for.

$$\begin{array}{r} 4 \\ 8\,\overline{)\,39} \\ -32 \\ \hline 7 \end{array}$$

4 boxes of 8 bags of candies in each

&

7 bags of ~~8~~ candies and 5 candies leftover

$(475)_8 = (317)_{10}$

# of candies in each bag

# of bags in each box

box — bag — loose candies

box — bag — loose candies

BASE – 10

Thousand	Hundred	Ten	One

BASE-8

Octhond	octred	oct	one

1 Counting in base Eight

1, 2, 3, 4, 5, 6, 7, 10, 11, ..., 17, 20, ...70, ..77, 100

Problem 5. *Use the chart below to make tally marks, write each number in words in base eight, and then write each number in digits in base eight. In the last row, don't actually make the tally marks for 128, but just indicate what you would do based on your tally marks for 64.*

Base Ten number	Tally Marks	Number in Words in Base Eight	Digits in Base Eight
7	|||||||	Seven	$(7)_8$
12	~~||||||||~~ ||||	One oct four	$(14)_8$
16	~~||||||||~~ ~~||||||||~~	Two octs	$(20)_8$
33	~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ |	Four octs one	$(41)_8$
62	~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ||||||	seven octs six	$(76)_8$
64	~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~ ~~||||||||~~	oct octs = one octred	$(100)_8$
128	double previous	two octreds	$(200)_8$
512	octmany times of tallies in 64	oct octreds = one octhond	$(1000)_8$

Problem 6. *Count 3 on to the following numbers. Actually do the counting! Record your result both in words and in digits.*

a) Two oct one, $(21)_8$

$(22)_8$, $(23)_8$, $(24)_8$

Two octs four

b) Three oct five, $(35)_8$

$(36)_8$, $(37)_8$, $(40)_8$

Four octs

c) Seven oct six, $(76)_8$

$(77)_8$, $(100)_8$, $(101)_8$

one octred one

Problem 7. *Count* **backward** *by 2 from the following numbers. Actually do the counting! Record your result both in words and in digits.*

a) One oct four, $(14)_8$

$(13)_8$, $(12)_8$

one oct two

b) Three oct, $(30)_8$

$(27)_8$, $(26)_8$

two octs six

c) Seven oct one, $(71)_8$

$(70)_8$, $(67)_8$

six oct seven

Problem 8. *Count* **backward** *by 4 from the following numbers. Actually do the counting! Record your result both in words and in digits.*

a) Three oct four, $(34)_8$

$(33)_8$, $(32)_8$, $(31)_8$, $(30)_8$

three octs

b) One oct two, $(12)_8$

$(11)_8$, $(10)_8$, $(7)_8$, $(6)_8$

six

c) Two octred Two, $(202)_8$

$(201)_8$, $(200)_8$, $(177)_8$, $(176)_8$

one octred six

Problem 9. *Start at* $(47)_8$ *and count up to the following numbers. Record your counting by listing the numbers written in digits in base 8. Your list should start with the number after* $(47)_8$*, and the last number on your list should be the number up to which you are counting. Then write,* **in base eight,** *how many numbers you counted from* $(47)_8$ *up to each number in the leftmost column of the chart.* **Be sure to answer in base 8.** *Think of strategies, and what would be better than listing all numbers. Think of several possible strategies.*

Count up to	List of numbers in base eight when counting up to this number	How many numbers did you count from $(47)_8$ to this number?
$(50)_8$	$(50)_8$	$(50)_8 - (47)_8 = (1)_8$
$(60)_8$	$(50)_8, (51)_8, (52)_8, (53)_8, (54)_8, (55)_8, (56)_8, (57)_8, (60)_8$	$(60)_8 - (47)_8 = (11)_8$ ⊛
$(67)_8$	$(61)_8, (62)_8, (63)_8, (64)_8, (65)_8, (66)_8, (67)_8$	$(67)_8 - (47)_8 = (20)_8$ ⊛
$(76)_8$	$(70)_8, (71)_8, (72)_8, (73)_8, (74)_8, (75)_8, (76)_8$	$(76)_8 - (47)_8 = (27)_8$ ⊛
$(100)_8$	$(77)_8, (100)_8$	$(100)_8 - (47)_8 = (31)_8$!!

⊛ $(47)_8 \xrightarrow{(1)_8} (50)_8 \xrightarrow{(10)_8} (60)_8$

⊛ $(47)_8 \xrightarrow{(10)_8} (57)_8 \xrightarrow{(10)_8} (67)_8$ $\boxed{\text{OR}}$ $(47)_8 \xrightarrow{(1)_8} (50)_8 \xrightarrow{(10)_8} (60)_8 \xrightarrow{(7)_8} (67)_8$

⊛ $(47)_8 \xrightarrow[(1)_8]{} (50)_8 \xrightarrow[(10)_8]{} (60)_8 \xrightarrow[(10)_8]{} (70)_8 \xrightarrow[(6)_8]{} (76)_8$

!! $(47)_8 \xrightarrow{(1)_8} (50)_8 \xrightarrow{(10)_8} (60)_8 \xrightarrow{(10)_8} (70)_8 \xrightarrow{(10)_8} (100)_8$

Problem 10. *When we count by threes in base ten we count "three, six, nine, twelve, etc." Can you count by threes in base eight? Starting at* $(3)_8$ *count by threes to just past octred.*

1. *List these numbers in base eight.*

$(3)_8$, $(6)_8$, $(11)_8$, $(14)_8$, $(17)_8$, $(22)_8$, $(25)_8$, $(30)_8$, $(33)_8$, $(36)_8$, $(41)_8$, $(44)_8$,

$(47)_8$, $(52)_8$, $(55)_8$, $(60)_8$, $(63)_8$, $(66)_8$, $(71)_8$, $(74)_8$, $(77)_8$, $(102)_8$

2. *Describe what patterns you notice in the numbers. How are these patterns similar to or different from counting by threes in base ten? Be sure think in base eight dont start thinking in base ten!*

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, ... Not very similar

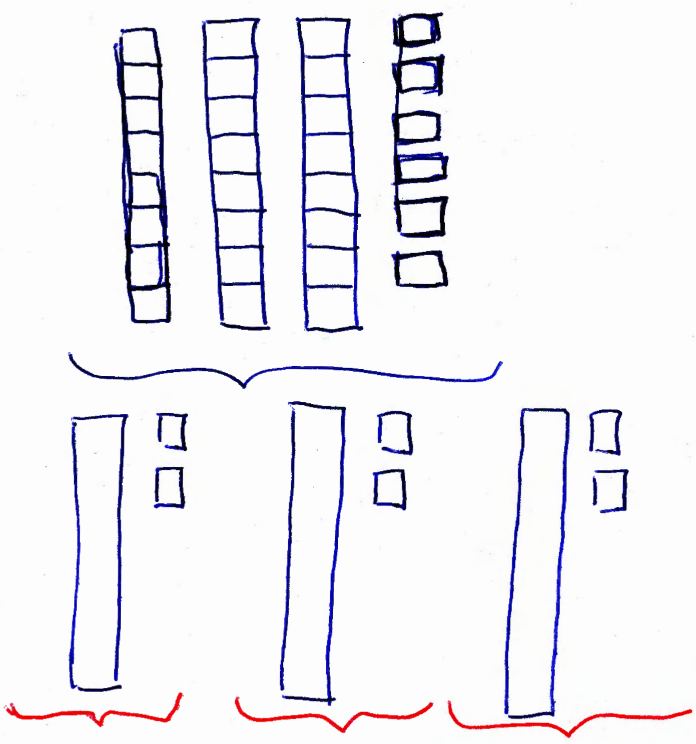
How about counting by 10s and 5s in base-10? Is it easier?

5, 10, 15, 20, 25, 30, ... 100 & 10, 20, 30, 40, ... 100

Is it similar to counting by $(10)_8$ and $(4)_8$ in base-8?

$(4)_8$, $(10)_8$, $(14)_8$, $(20)_8$, $(24)_8$, $(30)_8$, ... $(100)_8$ & $(10)_8$, $(20)_8$, $(30)_8$, $(40)_8$, ... $(100)_8$

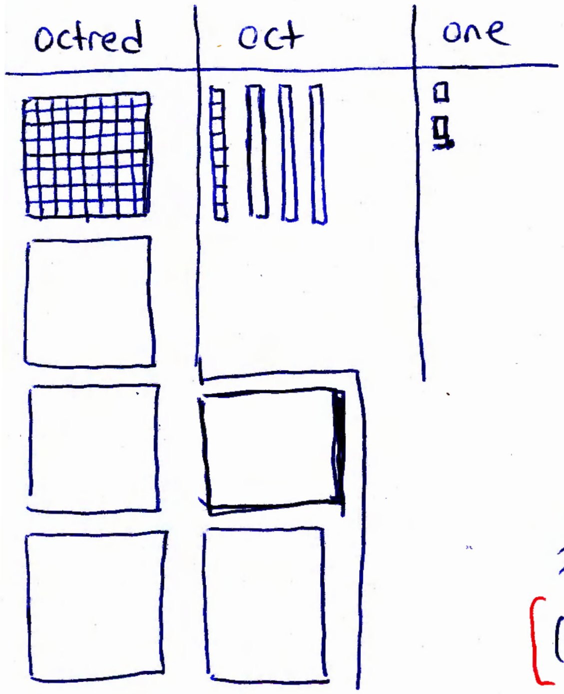
In part (1), you should have landed on $(36)_8$. *Explain why you have to land on this number when counting by threes, and use a picture in your explanation. Your explanation should not just state what this number means in base ten. Try to think in base eight, and utilize the picture(s).*



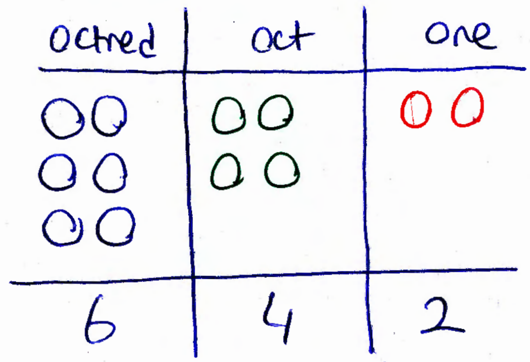
We landed on $(36)_8$ by counting by $(3)_8$'s because $(36)_8$ is divisible by $(3)_8$ in base-8.

Problem 11. *Present* $(642)_8$ *with base blocks, chip model, and write it in expanded form. (Read related base 10 material from 1.2, p.9)*

Base blocks



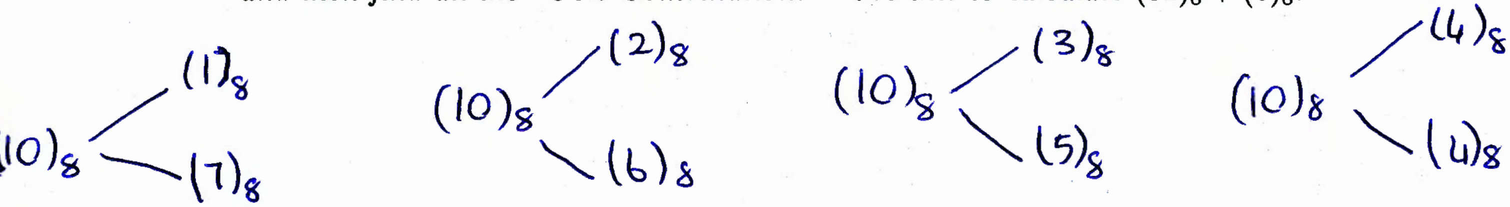
chip model



Expanded Form

$$\left[(6)_8 \times (100)_8\right] + \left[(4)_8 \times (10)_8\right] + \left[(2)_8 \times (1)_8\right] = (600)_8 + (40)_8 + (2)_8$$
$$= (642)_8$$

Problem 12. 1. *Look at p.11 in the book and pay attention to the "thens combinations", and then find all the "Octs Combinations". Use this to calculate* $(52)_8 + (6)_8$.



$$(52)_8 + (6)_8 = (50)_8 + (2)_8 + (6)_8$$
$$= (50)_8 + (10)_8 = (60)_8$$

2. *Calculate* $(70)_8 - (3)_8$. *Write your solution in several steps which, as above, make clear how to solve this using regrouping and an "octs combination".*

$$(70)_8 - (3)_8 = \left[(60)_8 + (10)_8\right] - (3)_8$$
$$= (60)_8 + \left[(10)_8 - (3)_8\right]$$
$$= (60)_8 + (5)_8$$
$$= (65)_8$$

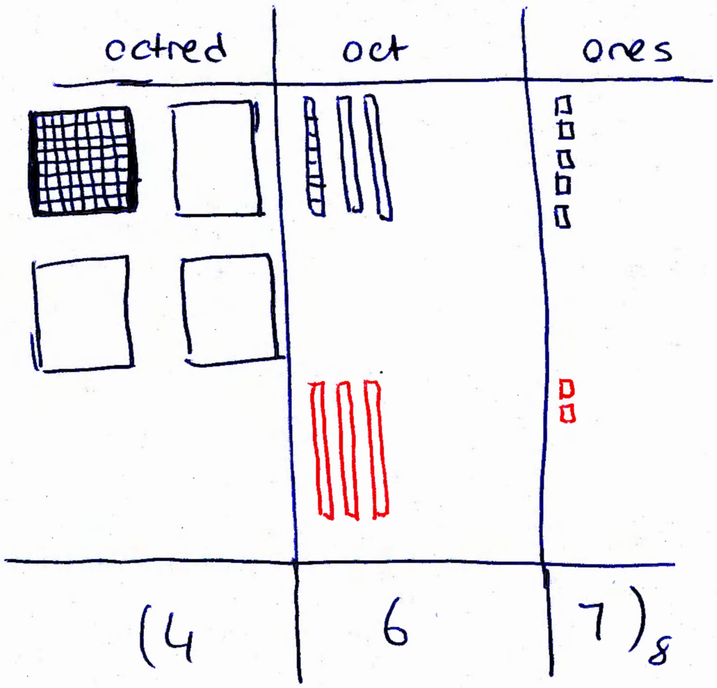
Problem 13. *Do the following operations in base eight, using a chip model and base blocks.*

1. $(435)_8 + (32)_8 =$

Chip model

Octred	Oct	One
○○○○	○○○	○○○○○
	○○○	○○
(4	6	7)$_8$

Base blocks



2. $(435)_8 - (32)_8 =$

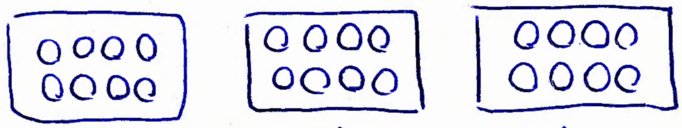
I will do chip model and leave base-blocks to students.

octred	oct	one
○○○○	~~○~~~~○~~~~○~~	○○○~~○~~ ~~○~~
○○○○	●	○○○
(4	0	3)$_8$

} no need to show this step

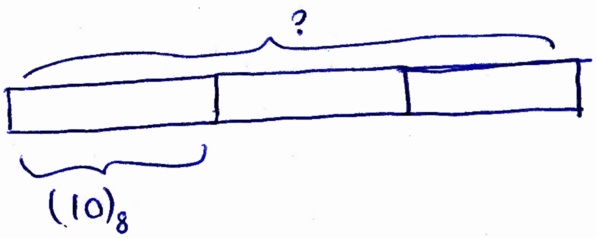
Problem 14. *(a) Illustrate the product $(3)_8 \times (10)_8$ using a set model, measurement model, and a rectangular array model as on p. 25 of your main text. For the array model use the version with the grid lines. Then illustrate the product $(10)_8 \times (10)_8$ using these models.*

Set model:

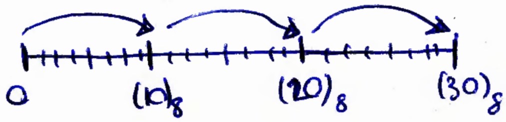


$(10)_8 + (10)_8 + (10)_8 = (3)_8 \times (10)_8 = (30)_8$

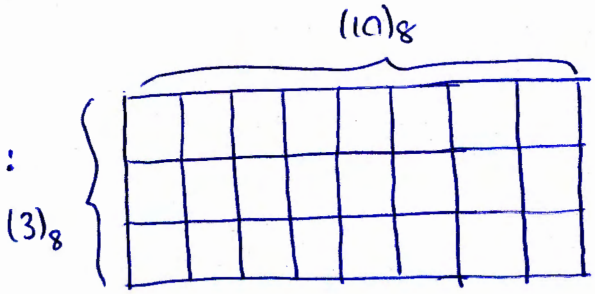
Measurement model:



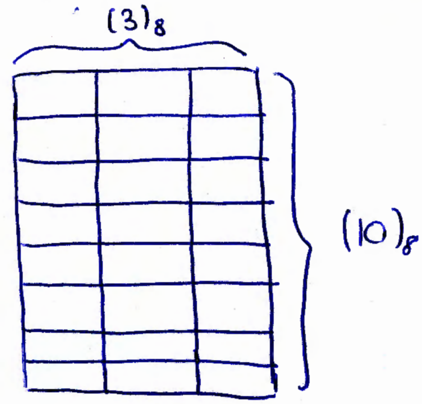
OR ⇒



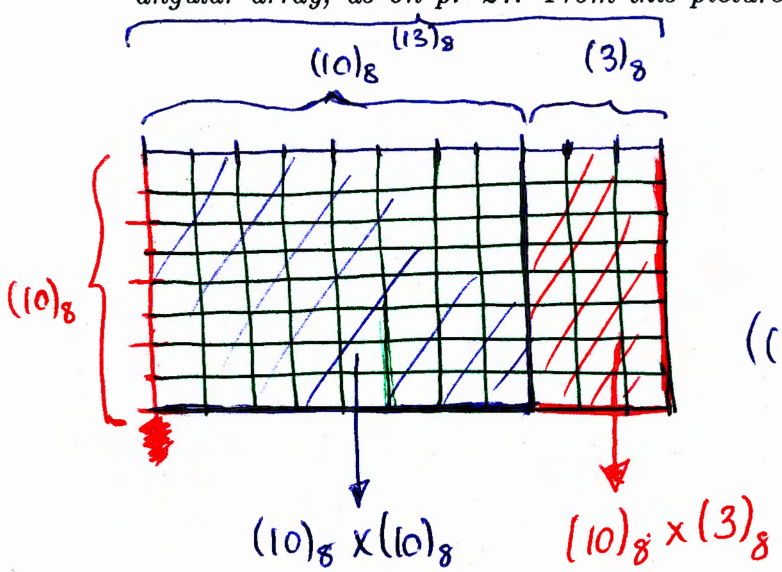
Rectangular Array model:



OR ⇒



(b) Illustrate the distributive property for the expression $((10)_8 + (3)_8)) \times (10)_8$ using a rectangular array, as on p. 27. From this picture, what do you obtain for $(13)_8 \times (10)_8$?



Total Area = The sum of partial areas

$((10)_8 + (3)_8) \times (10)_8 = (10)_8 \times (10)_8 + (10)_8 \times (3)_8$

Problem 15. *(a) Solve the following two division problems using the fact that* $(100)_8$ *is equal to oct octs:* $(100)_8 \div (4)_8 =$ $\underline{(20)_8}$ *and* $(100)_8 \div (2)_8 =$ $\underline{(40)_8}$.

(b) Based on your answers to part (a), find two pairs of compatible numbers (with respect to multiplication) in base 8. (See p. 44 for an explanation of compatible numbers in base 10.)

$$\left[(2)_8 \times (4)_8\right] \times (10)_8 = (10)_8 \times (10)_8 = (100)_8$$

✱ $(20)_8 \times (4)_8 = (100)_8$

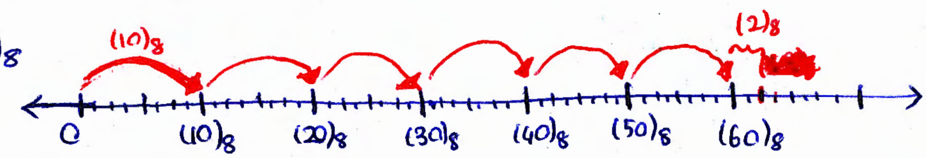
$(40)_8 \times (2)_8 = (100)_8$

Problem 16. *Illustrate the Quotient-Remainder theorem as specified. Parts (b),(c),(d) are in base 10. (Whenever there is no subscript indicating a different base, you should assume we are using base 10.)*

QRT ⇒ $\boxed{\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}}$

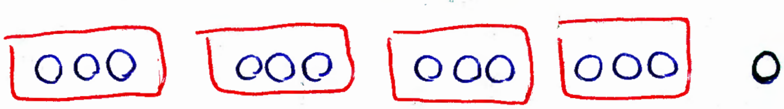
(a) A number line picture for $(62)_8 \div (10)_8$

$(62)_8 = \left[(10)_8 \times (6)_8\right] + (2)_8$



(b) A set model, using measurement division, *for* 13 ÷ 3 → the size of the group is known and division asks ✱ of groups

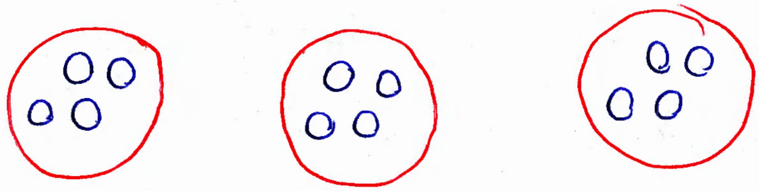
repeated subtraction of groups of 3



$(4 \times 3) + 1 = 13$

(c) A set model, using partitive division, *for* 13 ÷ 3 → ✱ of groups is known & division asks the size of the group

equal sharing / distribution among 3 people



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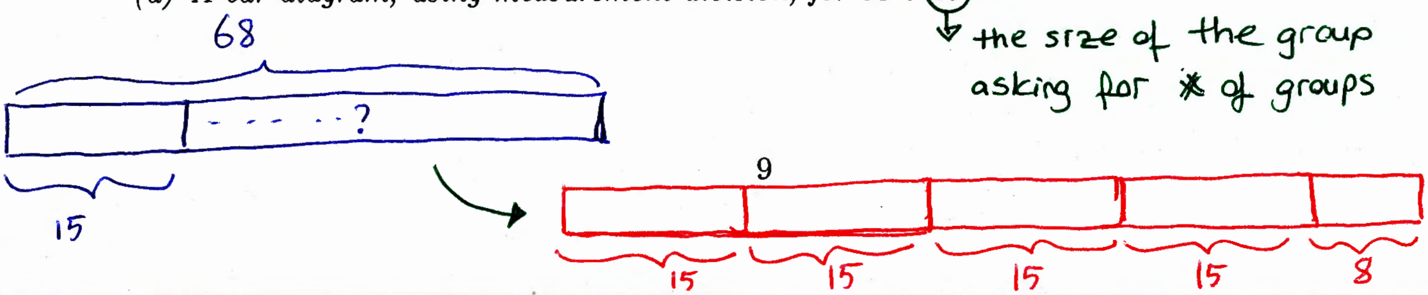
$(3 \times 4) + 1 = 13$

(d) A bar diagram, using measurement division, for 68 ÷ 15 → the size of the group asking for ✱ of groups

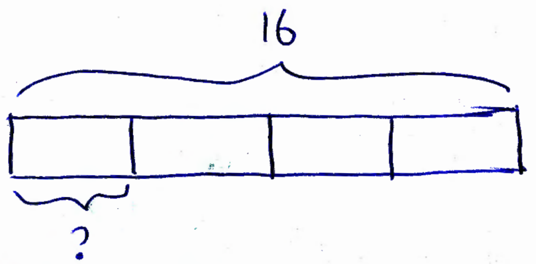


$(4 \times 15) + 8 = 68$

equal sharing

(e) A bar diagram, using partitive division, for $16 \div 4$ → # of groups

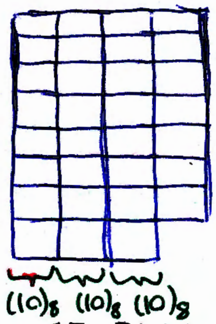
↳ asking for the size of each group



4 units = 16
1 unit = 4

$\div 4$

(f) A rectangular array model for $(37)_8 \div (10)_8$.



$$\left[(3)_8 \times (10)_8\right] + (7)_8 = (37)_8$$

$$(30)_8 + (7)_8 = (37)_8$$

Problem 17. *Division by zero:*

(a) What is $100 \div 0$*? Justify (prove) your answer.*

Assume that $100 \div 0$ is defined. Then there is a unique number a which satisfies $100 = 0 \times a$. (GRT)

Then $100 = 0$, which is not true.

So, there is no such a number a. $100 \div 0$ is undefined

(b) What is $0 \div 0$*? Justify (prove) your answer.*

Assume that $0 \div 0$ is defined. Then there is a unique number a which satisfies $0 = 0 \times a$, according to GRT.

Then $0 = 0$, which means that this statement is true for every number a. Contradiction because ~~[illegible]~~ GRT states that ~~[illegible]~~ there is a unique quotient (a). So, $0 \div 0$ is undefined.