Prealgebra
Activity 4
Math-T101 Spring 2014

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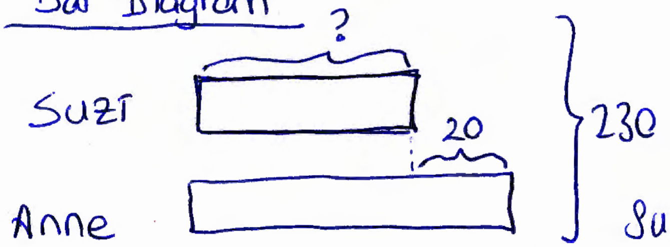
Problem 1. *(Algebraic teacher's solution) Suzi is 20 pounds lighter than Anne. Their total weight is 230 lbs. Find Suzi's weight.*

Bar Diagram



2 units = 230 - 20

2 units = 210

1 unit = 105

Suzi's weight is 105 lb.

Algebraic Solution:

x = Suzi's weight (lbs)

$x+20$ = Anne's weight (lbs)

$x + (x+20) = 230$

$2x+20 = 230$

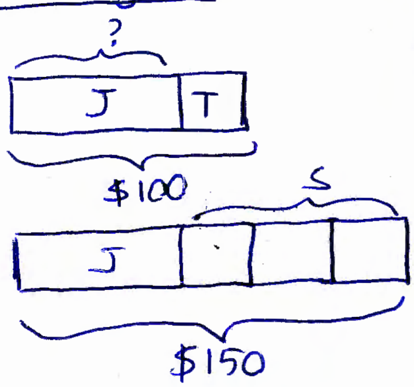
$2x = 230-20 \rightarrow 2x = 210$

$x = 105$

Suzi's weight is 105 lbs

Problem 2. *(Bar diagram, algebraic teacher's solution using only one variable and using multiple variables) Jenny, Tina, and Suzie went to the mall. Jenny and Tina spent a total of \$100 while Jenny and Suzie spent a total of \$150. If you know that Suzie spent 3 times as much as Tina did, then how much did Jenny spend?*

Bar Diagram



2 units = 150 - 100

2 units = 50

1 unit = 25

100 - 25 = 75

Jenny spent \$75.

Algebraic Solution using only one variable:

x = \$ Tina spent $\rightarrow$ $100-x$ = \$ Jenny spent

$3x$ = \$ Suzi spent $\rightarrow$ $150-3x$ = \$ Jenny.

$\boxed{100-x = 150-3x}$

$100-x = 150-3x$

$-100 \quad -100$

$-x = 50-3x$

$+3x \quad +3x$

$2x = 50$

$x = 25$

$100 - x = 100 - 25 = 75$

Jenny spent \$75

Algebraic solution using multiple variables:

x = \$ Tina spent

y = \$ Suzi spent

z = Jenny spent

$\boxed{x+z=100}$

$y+z = 150$

$\boxed{3x=y}$

$y+z=150$

$3x+z=150$

$2x+\boxed{x+z}=150$

$2x+100=150$

$-100 \quad -100$

$\rightarrow 2x=50$

$x=25$

$\rightarrow x+z=100$

$25+z=100$

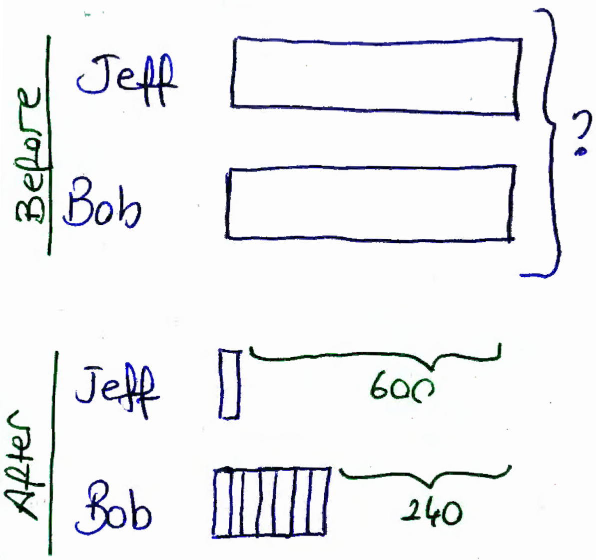
$-25 \quad -25$

$z=75$

Jenny spent \$75.

Problem 3. *(Bar diagram, algebraic teacher's solution using only one variable) Jeff and Bob began their day by splitting a bag of M & M's equally among themselves. Throughout the day Jeff ate 600 M & M's while Bob only ate 240. At the end of the day Bob observed that he still had 7 times as many M & M's as did Jeff. How many M & M's were in the original bag?*

Bar Diagram:



$600-240=360$

6 units $=360$

1 unit $=60$

$600+60=660$

$660+660=1320$

There were 1320 M&Ms in the original bag.

Algebraic Solution with one variable:

x = # of M&Ms Jeff had after eating 600 M&Ms

7x = # of M&Ms Bob had after eating 240 M&Ms

x+600 = # of M&Ms Jeff had at first

7x+240 = # of M&Ms Bob had at first

$$x+600 = 7x+240$$
$$-240 \qquad -240$$
$$x+360 = 7x$$
$$-x \qquad -x$$
$$360 = 6x$$
$$60 = x$$

because they splitted the original bag equally into two.

$60+600=660$

$7\cdot 60+240=660$

$660+660=1320$

There were 1320 M&Ms in the original bag.

order of operations: P, D - M, A - S

Problem 4. *Evaluate the following numerical expressions:*

a) $(8 \div 2) \times 4 = 4 \times 4 = 16$

b) $8 \div (2 \times 4) = 8 \div 8 = 1$

c) $8 \div 2 \times 4 = 4 \times 4 = 16$

d) $16 \div (4 \div 2) = 16 \div 2 = 8$

e) $(16 \div 4) \div 2 = 4 \div 2 = 2$

f) $16 \div 4 \div 2 = 4 \div 2 = 2$

g) $24 \div 4 + 2 = 6 + 2 = 8$

h) $24 + 6 \div 2 \times 3 = 24 + 3 \times 3 = 24 + 9 = 33$

Problem 5. *Use the identity* $(a+b)^2 = a_2 + 2ab + b^2$ *or* $(a-b)^2 = a_2 - 2ab + b^2$ *to calculate the following.*

a) $68^2 = (60+8)^2$
$= 60^2 + 2.60.8 + 8^2$
$= 3600 + 960 + 64 = 4624$

$68^2 = (70-2)^2$
$= 70^2 - 2.70.2 + 2^2$
$= 4900 - 280 + 4 = 4624$

b) $((27)_8)^2 = [(20)_8 + (7)_8]^2$
$= (20)_8^2 + 2.(20)_8.(7)_8 + (7)_8^2$
$= (400)_8 + (340)_8 + (61)_8$
$= (1021)_8$

$(27)_8^2 = [(30)_8 - (1)_8]^2$
$= (30)_8^2 - 2.(30)_8.(1)_8 + (1)_8^2$
$= (1100)_8 - (60)_8 + (1)_8 = (1021)_8$

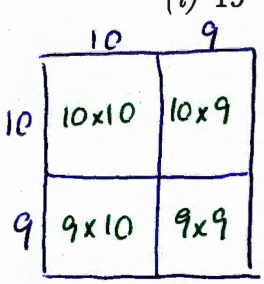
c) $121^2 =$

$121^2 = (120+1)^2 = 120^2 + 2.120.1 + 1^2$
$= 14400 + 240 + 1 = 14641$

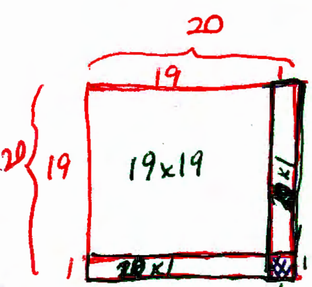
$121^2 = (130-9)^2 = 130^2 - 2.130.9 + 9^2$
$= 16900 - 2340 + 81 = 14641$

Problem 6. *Use a rectangular array model to find the following squares.*

(i) $19^2 = (10+9)^2 = (20-1)^2$

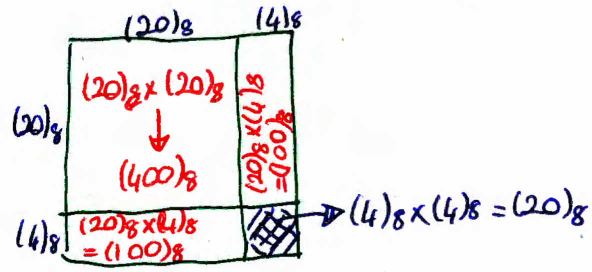


Total area $= 19 \times 19 = 19^2$
$= 10\times10 + 9\times10 + 9\times10 + 9\times9$
$= 100 + 90 + 90 + 81$
$= 10^2 + 2.9.10 + 9^2$
$= 361$



$19^2 = 20\times20 - 20\times1 - 20\times1 + 1\times1$
$= 20^2 - 2.20.1 + 1^2$
$= 400 - 40 + 1$
$= 361$

(ii) $((24)_8)^2 = [(20)_8 + (4)_8]^2$



Total Area → $(400)_8$
$(100)_8$
$(100)_8$
$+ (20)_8$
$(620)_8$

* Any order property: Commutative Property & Associative Property
$[a+b=b+a]$ $[(a+b)+c=a+(b+c)]$

(p. 99)

Problem 7. *Use the identity $(a+b)(a-b)=a^2-b^2$ to calculate the following.*

a) $28 \times 32 =$ $(30-2)\times(30+2)$

Find the average of 28 & 32 $\rightarrow \frac{28+32}{2} = 30$

$(30-2)\cdot(30+2) = 30^2-2^2$
$= 900-4$
$= 896$

b) $(27)_8 \times (31)_8 =$ $[(30)_8-(1)_8] \times [(30)_8+(1)_8] = (30)_8^2-(1)_8^2$

Find the average of $(27)_8$ & $(31)_8$ $\longrightarrow \frac{(27)_8+(31)_8}{2} = (30)_8$

$= (1100)_8 - (1)_8 = (1077)_8$

c) $108 \times 112 =$ $(110-2)\times(110+2)$ $\longrightarrow 110^2-2^2 = 12100-4$
$= 12096$

Find the average of 108 and 112 $\rightarrow \frac{108+112}{2} = \frac{220}{2} = 110$

Problem 8. *Which of the following are Algebraic Expression (AE), Equation (EQ), or Invalid (IN)?*

a) $5 \div 0$ IN
d) $x+y+=$ IN
g) $1+2=x$ EQ
b) $0 \div 0$ IN
e) $x^2+2ab=5$ EQ
c) x^2+2ab AE
f) $21 \div \times 7$ IN

Problem 9. *Show that $a^m \cdot a^n = a^{m+n}$ if a, m, and n are nonzero whole numbers.*

$a^m \cdot a^n = \underbrace{(a.a.a.\ldots.a)}_{m \text{ factors}} \cdot \underbrace{(a.a.a\ldots a)}_{n \text{ factors}}$ by defn. of exponents

$= \underbrace{(a.a.a.\ldots\ldots\ldots.a)}_{m+n \text{ factors}}$ by any order property $\rightarrow = a^{m+n}$ by defn. of exponents

Problem 10. *Show that $\frac{a^m}{a^n} = a^{m-n}$ if a, m, and n are nonzero whole numbers with $m > n$.*

$\frac{a^m}{a^n} = \frac{\overbrace{(a.a.a.\ldots\ldots a)}^{m \text{ factors}}}{\underbrace{(a.a.\ldots\ldots a)}_{n \text{ factors}}}$ by defn of exponens $= \frac{\overbrace{(a.a.\ldots.a)}^{n \text{ factors}} \cdot \overbrace{(a.a.\ldots a)}^{m-n \text{ factors}}}{\underbrace{(a.a.\ldots.a)}_{n \text{ factors}}}$ by any order property

$= \underbrace{(a.a.\ldots a)}_{m-n \text{ factors}}$ by canceling out n factors of a

a^{m-n} (by defn of exp.)

Problem 11. *Show that $(a^m)^n = a^{mn}$ if a, m, and n are nonzero whole numbers.*

$(a^m)^n = \underbrace{(a.a.\ldots.a)}_{m \text{ factors}}{}^n = \underbrace{\underbrace{(a.a.\ldots a)}_{m \text{ factors}} \cdot \underbrace{(a.a.\ldots a)}_{m \text{ factors}} \cdot \ldots \cdot \underbrace{(a.a.\ldots a)}_{m \text{ factors}}}_{n \text{ factors}}$ by defn of exponents

$= \underbrace{a.a.a.\ldots\ldots\ldots\ldots.a}_{m \times n \text{ factors}}$ by any order property

$= a^{m \times n} = a^{mn}$ by defn of exponents

Problem 12. *Show that $a^m \cdot b^m = (ab)^m$ if a, b, and m are nonzero whole numbers.*

$a^m \cdot b^m = \underbrace{(a.a....a)}_{m \text{ factors}} \cdot \underbrace{(b.b....b)}_{m \text{ factors}}$ by defn of exponents

$= \underbrace{(ab).(ab).............(a.b)}_{m \text{ factors}}$ by any-order property

$= (ab)^m$ by defn of exponents

Problem 13. *Show that $\frac{a^m}{a^n} = a^{m-n}$ if a, m and n are whole numbers and $m \neq n$. Note: If a is a nonzero whole numbers, then a^{-k} is defined to be $\frac{1}{a^k}$.*

$\frac{a^m}{a^n} = a^{m-n}$ if $m \neq n$, then $m > n$, OR $m < n$

see problem 10

work on this to show $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$

Special case: if $m=0, n=k$

$a^{-k} = a^{0-k} = \frac{a^0}{a^k} = \frac{1}{a^k}$

Problem 14. *The above rules can be extended to the case in which m and n are allowed to be 0 (but a and b are still nonzero). If a is a nonzero whole number, how should a^0 be defined? Justify this by the natural extensions of previous problems (rules).*

If $m=n$, then $1 = \frac{a^m}{a^n} = \frac{a^m}{a^m} = 1$

$= a^{m-m}$

$= a^0$

$\boxed{1 = a^0}$

$a^0 . a^m = a^{0+m} = a^m \iff a^0 = 1$

$(a^0)^m = a^{0.m} = a^0 \iff a^0 = 1$

$1 = (ab)^0 = a^0 . b^0 = 1.1 = 1$

Some other properties justifying $a^0 = 1$

Problem 15. *Calculate the following mentally using $2^5 = 32$, $2^8 = 256$, and $2^{10} = 1024$.*

a) $1024 \div 256 = \frac{2^{10}}{2^8} = 2^2 = 4$

b) $64 \times 128 = 2^6 . 2^7 = 2^{13}$

c) $2048 \div 256 \times 16 = \frac{2^{11}}{2^8} . 2^4$

$= 2^{11-8} . 2^4$

$= 2^3 . 2^4 = 2^{3+4} = 2^7$

d) $(2^3)^5 \div 2^9 = \frac{2^{15}}{2^9} = 2^{15-9} = 2^6$

e) $8^5 \div 512 = \frac{(2^3)^5}{2^9} = \frac{2^{15}}{2^9} = 2^{15-9} = 2^6$

f) $256 \times 5^3 = 2^8 \times 5^3 = 2^5 . \underbrace{2^3 . 5^3} = 2^5 . 10^3 = 32 . 10^3 = 32 . 1000 = 32000$

g) $80^3 = 8^3 . 10^3 = (2^3)^3 . 2^3 . 5^3 = 2^9 . 2^3 . 5^3 = 2^{12} . 5^3$

Problem 16. *Let a and b be non-zero whole numbers. Simplify as much as possible, factoring the numbers and leaving the answer in the exponential form.*

a) $\frac{2^5 \cdot 6^2 \cdot 18^2}{3^4 \cdot 4^2} = \frac{2^5 . (2 . 3)^2 . (2 . 3^2)^2}{3^4 . (2^2)^2} = \frac{2^5 . 2^2 . 3^2 . 2^2 . (3^2)^2}{3^4 . 2^4} = \frac{2^5 . \cancel{2^2} . 3^2 . \cancel{2^2} . \cancel{3^4}}{\cancel{3^4} . \cancel{2^4}}$

$= 2^5 . 3^2$

b) $\frac{2^5 \cdot (2b)^2 \cdot (2b^2)^2}{b^4 \cdot 4^2} = \frac{2^5 . 2^2 b^2 . 2^2 (b^2)^2}{b^4 . (2^2)^2} = \frac{2^9 . b^2 . \cancel{b^4}}{\cancel{b^4} . 2^4} = 2^{9-4} . b^2$

$= 2^5 . b^2$

c) $\frac{a^5 \cdot (ab)^2 \cdot (ab^2)^2}{b^4 \cdot (a^2)^2} = \frac{a^5 . a^2 . b^2 . a^2 . (b^2)^2}{b^4 . (a^2)^2} = \frac{a^5 . \cancel{a^2} . b^2 . \cancel{a^2} . \cancel{b^4}}{\cancel{b^4} . \cancel{a^4}}$

$= a^5 . b^2$